

# Hadron Spectroscopy

Classification and phenomenology



Mikhail Mikhasenko · Lecture 1  
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# Lecture 1 roadmap

- A. QCD basics**
- B. Excited states**
- C. Observation of hadrons**
- D. Decays and the hadronic continuum**

# QCD basics

A · Fields, vacuum and confinement

# The QCD Lagrangian

[PDG: QCD \(2025\)](#)

[PDG: Quark model \(2025\)](#)

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}$$

**Quarks carry colour**

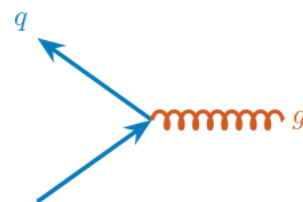
$$D_\mu = \partial_\mu - ig_s T^a A_\mu^a$$

$q_f$ : quark of flavour  $f$ ;  $T^a$ : colour generators.

**Gluons interact with gluons**

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$$

Eight gluon fields,  $a = 1, \dots, 8$ .



quark-gluon



three gluons



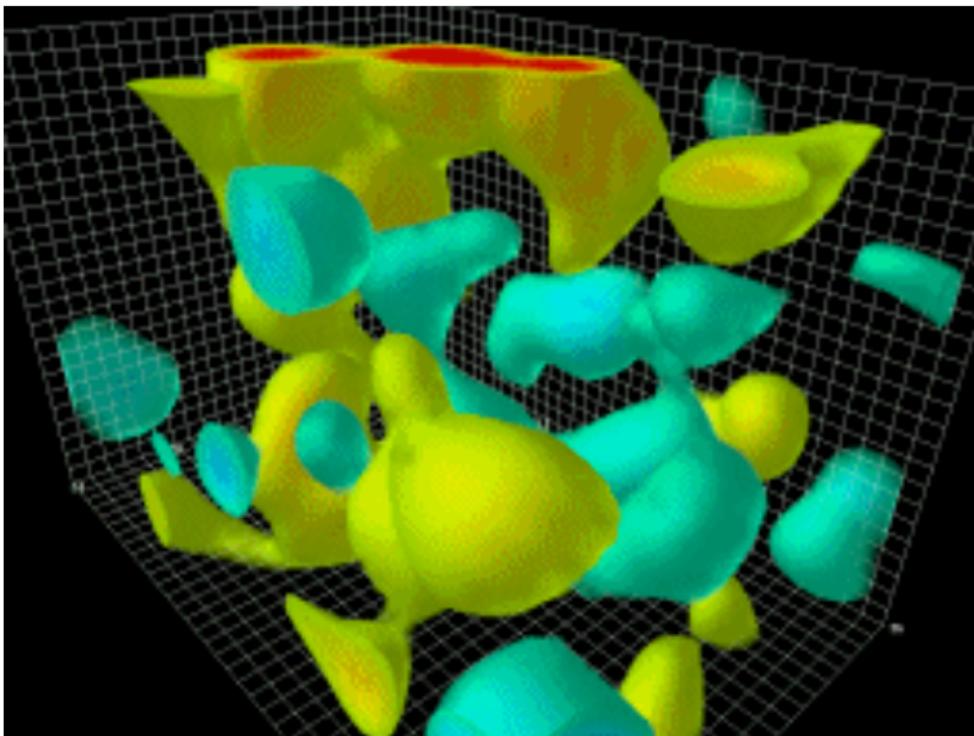
four gluons

The non-Abelian term generates the three- and four-gluon vertices. Gauge-fixing and ghost terms are omitted here.

# The QCD vacuum

[Derek Leinweber: original animations](#)

[PDG: QCD \(2025\)](#)



The QCD Lava Lamp · Derek Leinweber, Adelaide

A view of a smoothed quantum field, not individual gluons flying through empty space. The animation is not real-time motion.

## Empty space is not empty of fields.

Even the lowest-energy state contains fluctuating chromo-electric and chromo-magnetic fields.

## What are we looking at?

- A lattice-QCD field in a box of  $2.4 \times 2.4 \times 3.6 \text{ fm}^3$  — a few proton sizes.
- **50 smoothing sweeps** reveal the larger-scale lumps.
- Colour shows **topological charge density**: the local sign and strength of the field's winding.

| Run on your computer: [Akio Tomiya's VisualizingLQCD.jl](#)

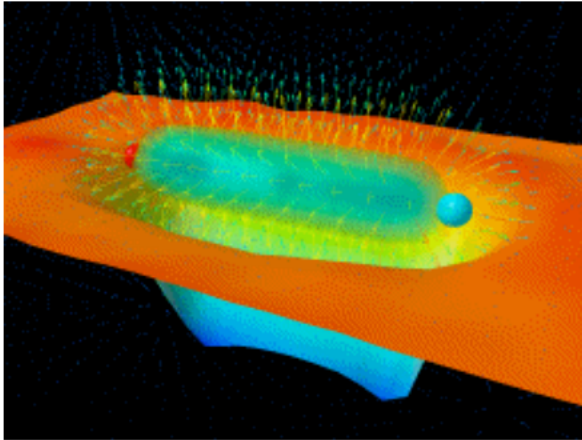
# Confinement and flux tubes

[Leinweber: original animations](#)

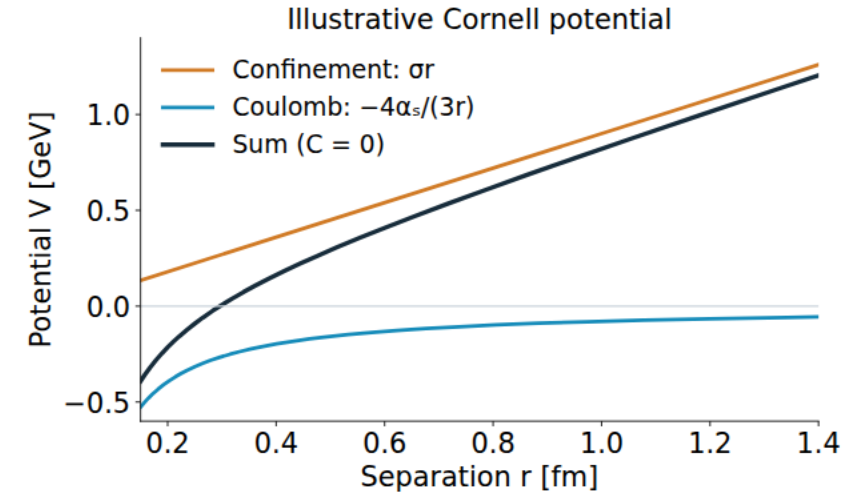
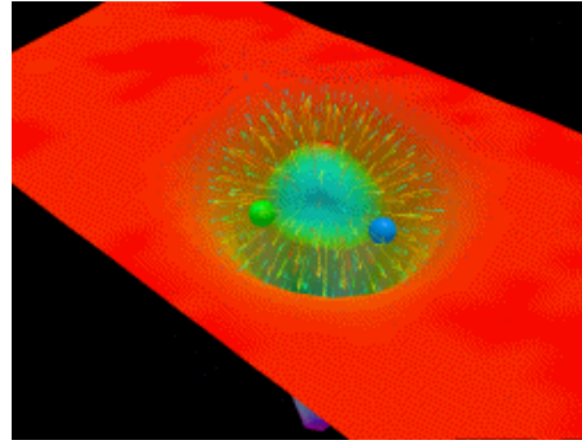
[Bissey et al., PRD 76 \(2007\) 114512](#)

[Biddle et al.: vortices and the potential](#)

Quark–antiquark:  $q\bar{q}$



Three quarks:  $qqq$



## Pull the quarks apart

The field forms a longer tube of almost constant width. More length costs more energy.

Surfaces: reduction of vacuum action density around static sources.

$$V(r) = \underbrace{-\frac{4\alpha_s}{3r}}_{\text{short-range attraction}} + \underbrace{\sigma r}_{\text{energy of the tube}} + C$$

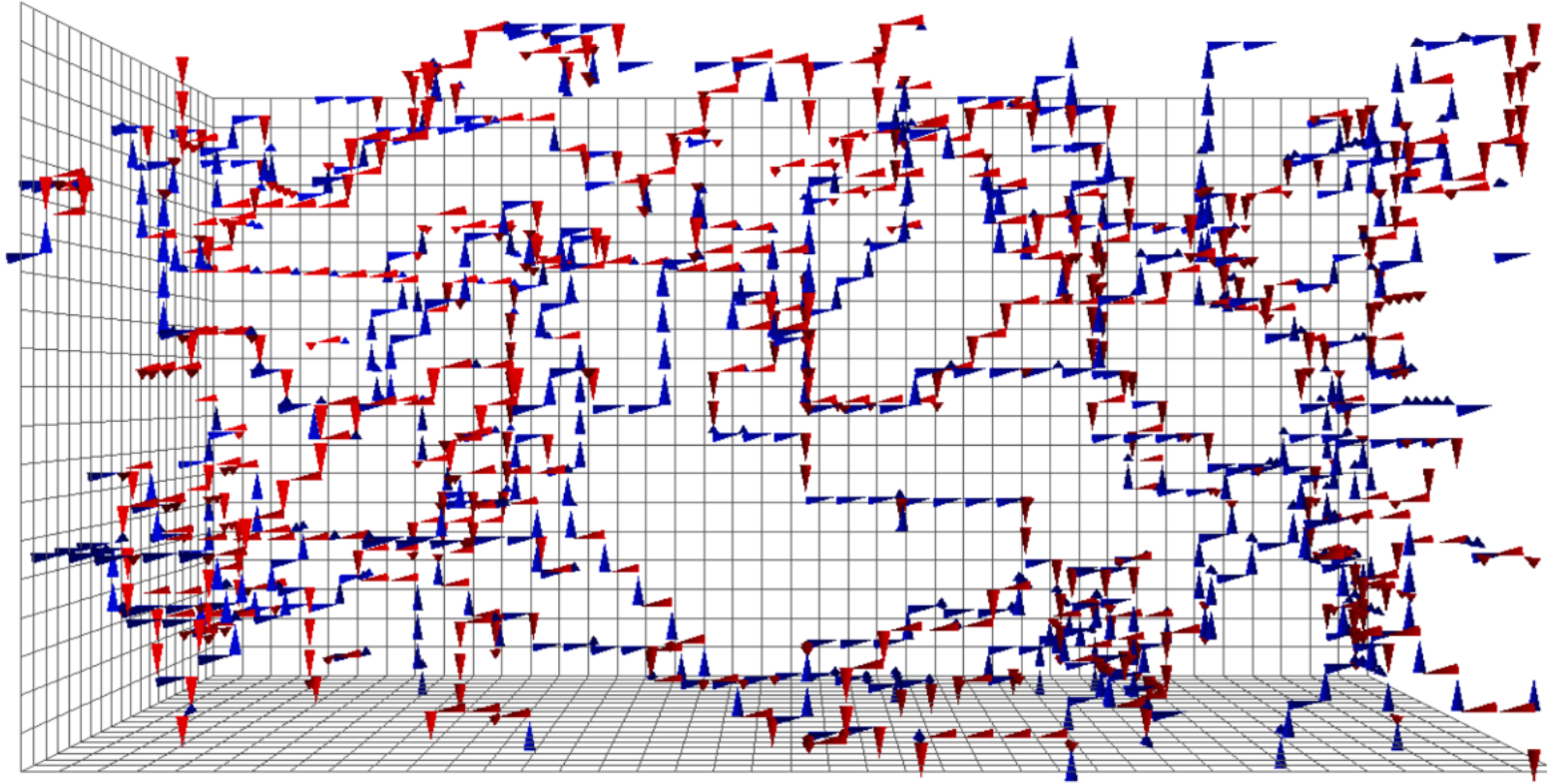
$C$  sets the energy zero. With light quarks, a long string can **break into two hadrons**.

What field structures support confinement? Lattice studies link the linear potential to **centre vortices**.

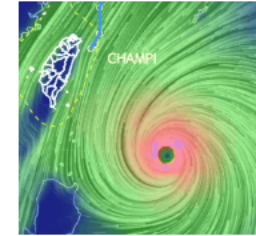
# Centre vortices

[Biddle, Kamleh & Leinweber, PRD 102 \(2020\) 034504](#)

[Biddle et al.: static potential \(2022\)](#)



A 3D slice through the vortex network · Biddle, Kamleh & Leinweber



A familiar vortex — an analogy, not a QCD simulation.

## Read the map

Lines mark centre flux; red and blue distinguish its orientation.

A 3D slice cuts the four-dimensional vortex surfaces into lines.

Remove the identified vortices in a lattice calculation → the confining linear rise disappears. Keep them → the confining behaviour survives.

# Effective degrees of freedom

## Gluons: self-interactions and confinement

## Light quarks: $u, d, s$

- **Light:**  $u, d$  nearly massless on hadronic scales;  $s$  is heavier.
- **Cheap:** light quark–antiquark pairs are easier to create.
- **Mixed:** neutral light mesons can mix flavour components.

## Heavy quarks: $c, b$ ( $t$ )

- **Massive:** charm and bottom provide a large mass scale.
- **Expensive:** creating a heavy pair needs much more energy.
- **Persistent:** a heavy quark usually remains a useful label for its hadron.

Strong interactions conserve net flavour for all quarks.  
The top quark decays before forming ordinary hadrons.

| I | II | III |
|---|----|-----|
| u | c  | t   |
| d | s  | b   |

$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - ig_{cw} (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\mu^0 (W_\mu^+ \partial_\nu W_\nu^- - W_\nu^- \partial_\mu W_\mu^+) + Z_\mu^0 (W_\mu^+ \partial_\nu W_\nu^- - W_\nu^- \partial_\mu W_\mu^+)) - \\
 & ig_{sw} (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\mu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - \\
 & W_\nu^- \partial_\mu W_\mu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - \\
 & Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w (A_\mu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
 & \beta_h \left( \frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \\
 & \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 & g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \\
 & \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
 & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
 & M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- - W_\mu^- \partial_\mu \phi^+) - ig \frac{s_w}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\
 & W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 & \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 & \frac{1}{2}g^2 \frac{s_w}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2}ig_s \lambda_{ij}^2 (\bar{q}_i^c \gamma^\mu q_j^c) g_\mu^a - \bar{e}^\lambda (\gamma^\mu + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma^\mu + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma^\mu + \\
 & m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma^\mu + m_d^\lambda) d_j^\lambda + ig s_w A_\mu (-\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda) + \\
 & \frac{ig}{4c_w} Z_\mu^0 \{ (\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
 & (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 + \gamma^5) u_j^\lambda) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}_{\lambda\kappa} e^\kappa) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)) + \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep\dagger}_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\kappa\lambda}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\lambda (\bar{\nu}^\lambda U^{lep}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_e^\lambda (\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa} (1 + \gamma^5) \nu^\kappa) - m_\nu^\kappa (\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa} (1 - \gamma^5) \nu^\kappa) - \frac{g}{2} \frac{m_\nu^\lambda}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda^\lambda}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\lambda^\lambda}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\lambda^\lambda}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \bar{\nu}_\kappa - \\
 & \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \bar{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_\lambda^\lambda}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda^\lambda}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\lambda^\lambda}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\lambda^\lambda}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c + \\
 & \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + ig_{cw} W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
 & \partial_\mu \bar{X}^- X^0) + ig_{sw} W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^- Y) + ig_{cw} W_\mu^- (\partial_\mu \bar{X}^0 X^+ - \\
 & \partial_\mu \bar{X}^+ X^0) + ig_{sw} W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + ig_{cw} Z_\mu^0 (\partial_\mu \bar{X}^+ X^- - \\
 & \partial_\mu \bar{X}^- X^+) + ig_{sw} A_\mu (\partial_\mu \bar{X}^+ X^- - \\
 & \partial_\mu \bar{X}^- X^+) - \frac{1}{2}gM (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H) + \frac{1-2c_w^2}{2c_w} igM (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
 & \frac{1}{2c_w} igM (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + igM s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
 & \frac{1}{2}igM (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
 \end{aligned}$$

M. Veltman, [Diagrammatica](#)

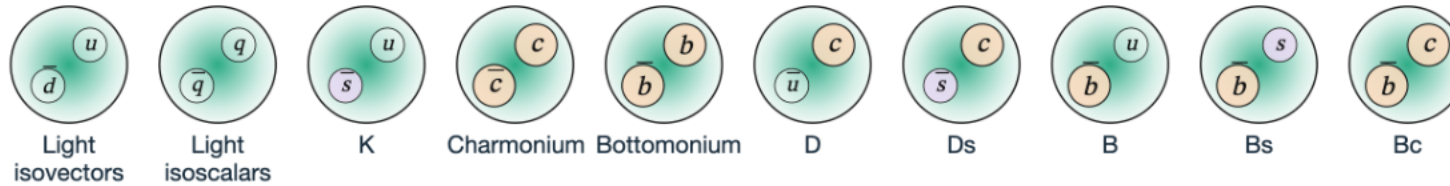
# Quarks and hadron families

[PDG: Quark model \(2025\)](#)

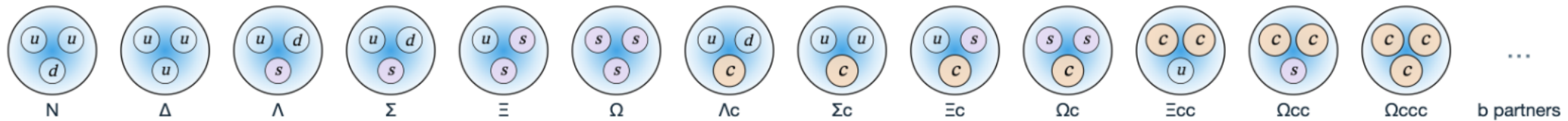
[PDG: Light mesons \(2025\)](#)

[LHCb: doubly-charmed baryons \(2026\)](#)

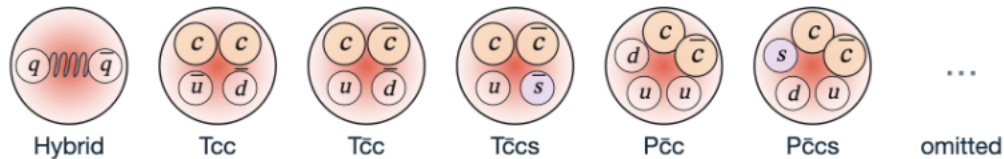
## Mesons · $q \bar{q}$



## Baryons · $q q q$



## Exotic configurations



Flavour chooses the family; **spin and orbital motion** generate its states.

Categories group charge partners and antiparticles; this is not a count of individual particles.

# Excited states

B · Spin, orbital motion and excitation multiplets

# Radial and orbital multiplets

[Godfrey-Isgur, PRD 32 \(1985\) 189](#)

[GIModel.jl: wavefunctions](#)

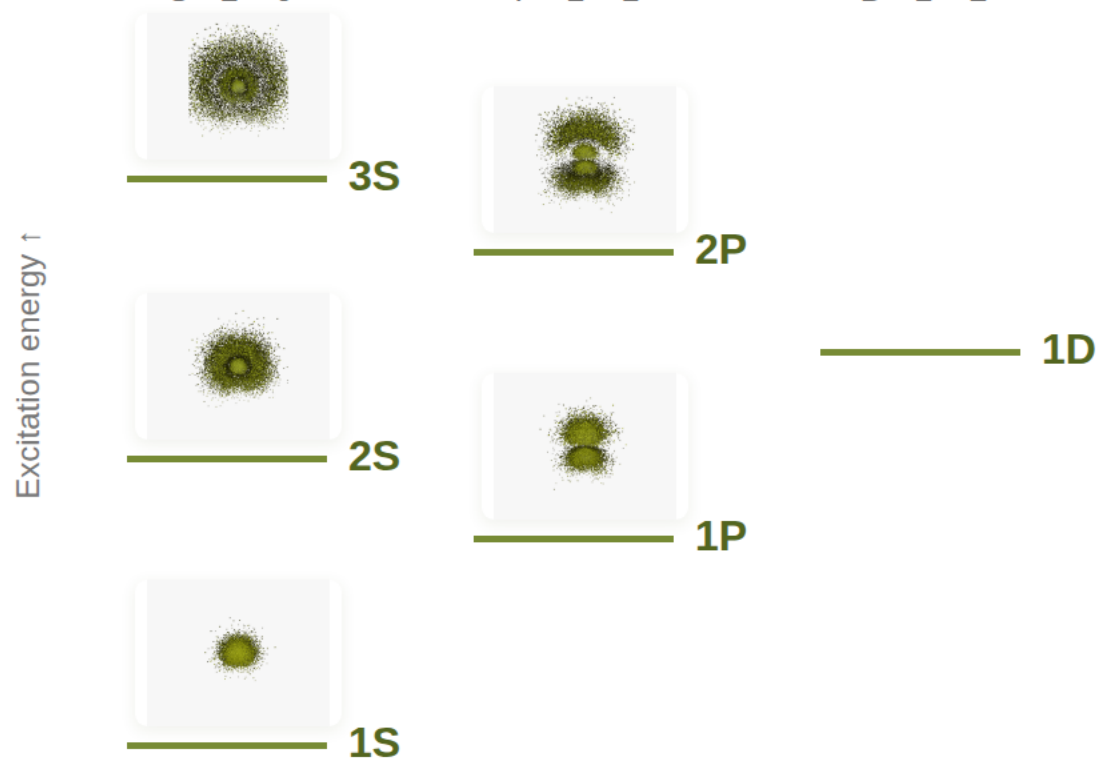
## Wavefunctions and nodes

$$H\psi = E\psi$$

$$S \cdot L = 0$$

$$P \cdot L = 1$$

$$D \cdot L = 2$$



GIModel charmonium clouds · schematic energy spacings

## Radial excitation

- Same orbital angular momentum  $L$ .
- More nodes in the radial wavefunction.
- $1S \rightarrow 2S \rightarrow 3S$ : **0** → **1** → **2 radial nodes**.

## Orbital excitation

- Increase  $L$ : **S** → **P** → **D**.
- Change the angular shape of the wavefunction.
- The displayed P waves have an angular nodal plane.

## Connection to hydrogen

$N = n_r + L + 1$ , while our radial label is  $n = n_r + 1$ .

Confinement lifts the Coulomb degeneracy: **1P and 2S need not coincide**.

# Spin and spin projection

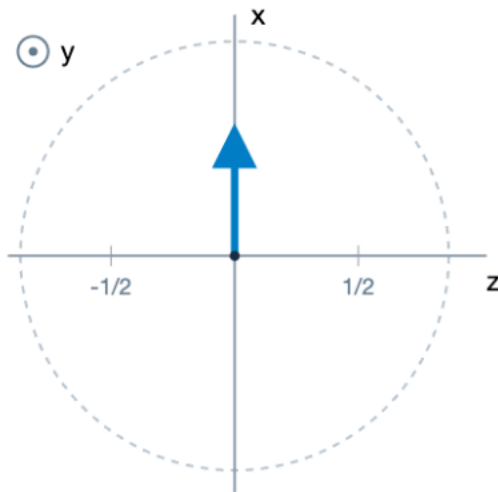
[PDG: CG coefficients \(2025\)](#)

[PDG: Quark model \(2025\)](#)

$$\mathbf{j}^2 |jm\rangle = \hbar^2 j(j+1) |jm\rangle, \quad j_z |jm\rangle = \hbar m |jm\rangle.$$

Fixed **spin**  $j$ ;  $2j + 1$  possible projections:  $m = -j, -j + 1, \dots, j$ .

Spin  $j$      $\theta$    $90^\circ$



$$|\psi(\theta)\rangle = 0.707 |1/2, 1/2\rangle + 0.707 |1/2, -1/2\rangle$$

$$|c_m|^2 =$$

50.0%

50.0%

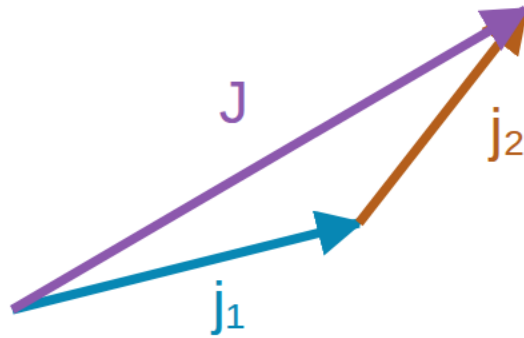
$$\sum |c_m|^2 = 1.000 \cdot 2 \text{ possible values of } m$$

# Adding two spins

[PDG: CG coefficients \(2025\)](#)

[PDG: Quark model \(2025\)](#)

## 1 · Add angular momenta



$$\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2.$$

Vector addition — with **quantized** total angular momentum.

## 2 · Find the allowed total spins

$$j_1 \otimes j_2 = |j_1 - j_2| \oplus \cdots \oplus (j_1 + j_2).$$

$J$  increases in steps of one. Each value labels a separate multiplet.

## 3 · Then resolve the projections

For each  $J$ , there are  $2J + 1$  states:

$$M = -J, \dots, J, \quad M = m_1 + m_2.$$

$$\underbrace{(2j_1 + 1)(2j_2 + 1)}_{\text{product states}} = \sum_{\underbrace{J}_{\text{coupled states}}} (2J + 1).$$

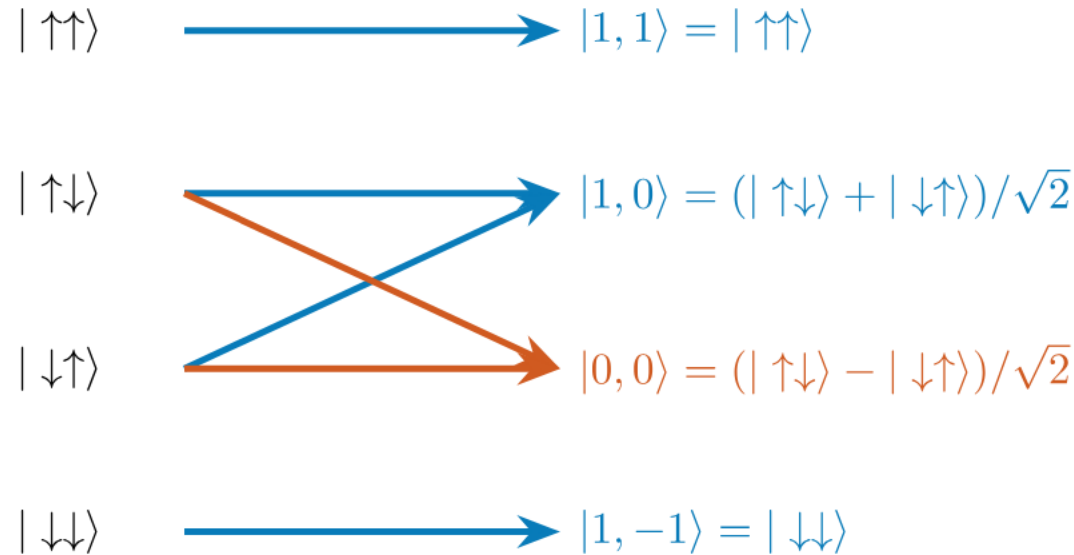
# Example: singlet and triplet

[PDG: CG coefficients \(2025\)](#)

[PDG: Quark model \(2025\)](#)

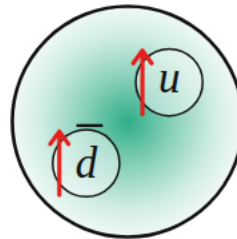
Product basis

Coupled basis



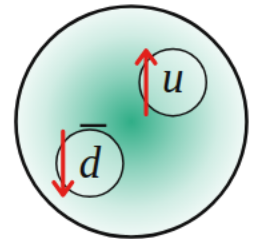
**Triplet  $S = 1$ :** symmetric under exchange.

Three projections:  $M = 1, 0, -1$ .



**Singlet  $S = 0$ :** antisymmetric under exchange.

One projection:  $M = 0$ .



# Spin and orbital motion

[PDG: CG coefficients \(2025\)](#)

[PDG: Quark model \(2025\)](#)

$$j_1^{P_1} \otimes j_2^{P_2} \rightarrow S^{P_1 P_2}, S'^{P_1 P_2}, \dots \quad (L = 0).$$

| $\frac{3}{2}^+ \otimes \frac{1}{2}^-$ | S = 1 ladder                                 | S = 2 ladder                                                               | Relative motion |
|---------------------------------------|----------------------------------------------|----------------------------------------------------------------------------|-----------------|
|                                       | <b>1<sup>-</sup></b>                         | <b>2<sup>-</sup></b>                                                       | S wave<br>L = 0 |
|                                       | 0 <sup>+</sup> 1 <sup>+</sup> 2 <sup>+</sup> | 1 <sup>+</sup> 2 <sup>+</sup> 3 <sup>+</sup>                               | P wave<br>L = 1 |
|                                       | 1 <sup>-</sup> 2 <sup>-</sup> 3 <sup>-</sup> | 0 <sup>-</sup> 1 <sup>-</sup> 2 <sup>-</sup> 3 <sup>-</sup> 4 <sup>-</sup> | D wave<br>L = 2 |
|                                       | 2 <sup>+</sup> 3 <sup>+</sup> 4 <sup>+</sup> | 1 <sup>+</sup> 2 <sup>+</sup> 3 <sup>+</sup> 4 <sup>+</sup> 5 <sup>+</sup> | F wave<br>L = 3 |

$$J = |L - S|, \dots, L + S, \quad P = P_1 P_2 (-1)^L.$$

Each initial  $S$  **generates its own orbital ladder**. Repeated  $J^P$  values in different columns are distinct coupled configurations.

# Meson quantum numbers

[PDG: Quark model \(2025\)](#)

[PDG: CG coefficients \(2025\)](#)

For a **neutral, self-conjugate**  $q\bar{q}$  state, charge conjugation exchanges quark and antiquark.

Parity:  $P = (-1)^{L+1}$

**Charge conjugation:  $C = (-1)^{L+S}$**

| $\frac{1}{2}^+ \otimes \frac{1}{2}^-$ | <b>S = 0 ladder</b> | <b>S = 1 ladder</b>        | <b>Relative motion</b> |
|---------------------------------------|---------------------|----------------------------|------------------------|
|                                       | $0^{-+}$            | $1^{--}$                   | S wave<br>$L = 0$      |
|                                       | $1^{+-}$            | $0^{++} \ 1^{++} \ 2^{++}$ | P wave<br>$L = 1$      |
|                                       | $2^{-+}$            | $1^{--} \ 2^{--} \ 3^{--}$ | D wave<br>$L = 2$      |
|                                       | $3^{+-}$            | $2^{++} \ 3^{++} \ 4^{++}$ | F wave<br>$L = 3$      |

**Forbidden for conventional  $q\bar{q}$ :**  $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$

# Isospin

[PDG: Quark model \(2025\)](#)

[PDG: CG coefficients \(2025\)](#)

**u** ↔ **d**

Nearly equal masses on the strong-interaction scale.

QCD treats them almost identically → **approximate isospin symmetry**.

## The same addition rules as spin

**One light quark** ( $u, d$ ) forms a doublet:

$$I = \frac{1}{2}, \quad I_3(u) = +\frac{1}{2}, \quad I_3(d) = -\frac{1}{2}.$$

**Three light quarks:**

$$\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{3}{2}.$$

**Nucleons:**  $I = \frac{1}{2} \rightarrow p, n$

**Delta baryons:**  $I = \frac{3}{2} \rightarrow \Delta^{++}, \Delta^+, \Delta^0, \Delta^-$

Here “light” means **u or d**. The  $s, c, b$  quarks are isospin singlets:  $I = 0$ .

## Neutral mesons: equal-weight mixtures

The  $u\bar{u}$  and  $d\bar{d}$  flavour configurations mix into states of **definite isospin**:

**Isoscalar** ·  $I = 0$

$$|0, 0\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle}{\sqrt{2}}.$$

**Isovector** ·  $I = 1$

$$|1, 0\rangle = \frac{|u\bar{u}\rangle - |d\bar{d}\rangle}{\sqrt{2}}.$$

**Maximal mixing:** equal magnitudes, with a **plus or minus relative sign**.

Nearly degenerate **members of each multiplet** — not necessarily equal masses for the  $I = 0$  and  $I = 1$  multiplets.

# Baryons: good and bad diquarks

PDG: Quark model (2025)  
Jaffe: Exotica, hep-ph/0409065

Heavy quark:  $s_Q = \frac{1}{2}$

Light pair: internal S wave, colour  $\bar{3}$

## Good diquark

$$j_d^P = 0^+$$

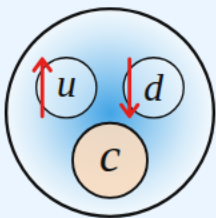
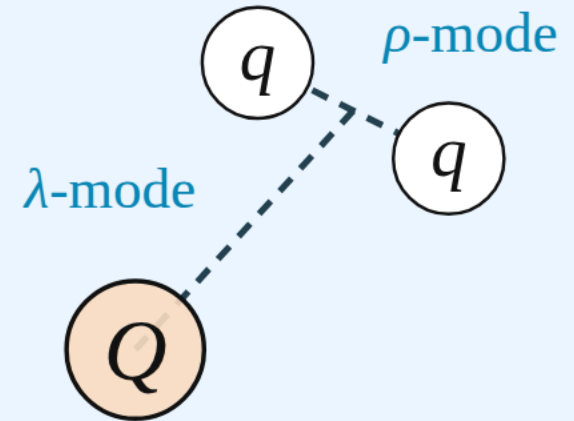
Antisymmetric flavour

Relative  $L = 0$ :  $\mathbf{J} = \mathbf{s}_Q + \mathbf{j}_d$ ,  $P = +$ .

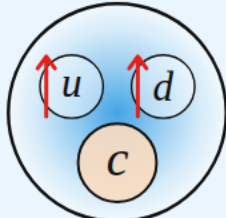
## Bad diquark

$$j_d^P = 1^+$$

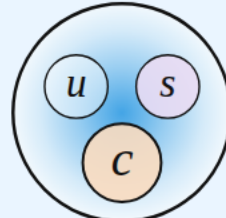
Symmetric flavour



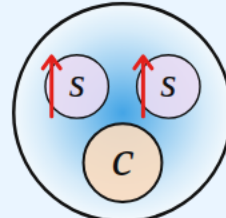
$\Lambda_c$  · good



$\Sigma_c$  · bad (isospin)



$\Xi_c, \Xi'_c$  · good + bad



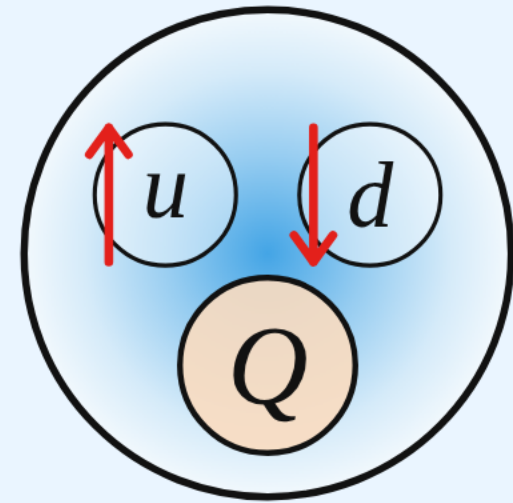
$\Omega_c$  · bad (identity)

# A heavy quark and a good diquark

[PDG: Quark model \(2025\)](#)  
[Jaffe: Exotica, hep-ph/0409065](#)

For  $Q[ud]$  with  $j_d^P = 0^+$  and relative  $L = 0$ , which  $J^P$  levels are allowed?

- A.  $\frac{1}{2}^+$  only
- B.  $\frac{3}{2}^+$  only
- C.  $\frac{1}{2}^+$ ,  $\frac{3}{2}^+$
- D.  $\frac{1}{2}^-$  only

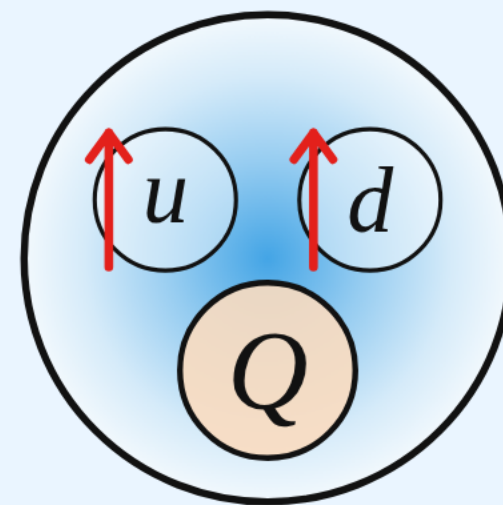


# A heavy quark and a bad diquark

[PDG: Quark model \(2025\)](#)  
[Jaffe: Exotica, hep-ph/0409065](#)

For  $Q\{ud\}$  with  $j_d^P = 1^+$  and relative  $L = 0$ , which  $J^P$  levels are allowed?

- A.  $\frac{1}{2}^+$  only
- B.  $\frac{3}{2}^+$  only
- C.  $\frac{1}{2}^+$ ,  $\frac{3}{2}^+$
- D.  $\frac{1}{2}^-$ ,  $\frac{3}{2}^-$



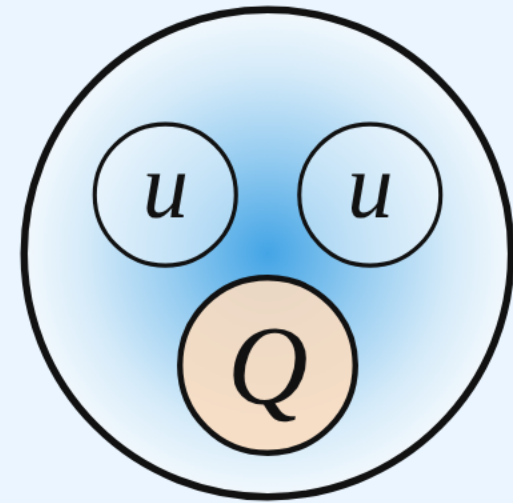
# Can the good diquark be $uu$ ?

[PDG: Quark model \(2025\)](#)

[Jaffe: Exotica, hep-ph/0409065](#)

For an internally S-wave, colour- $\bar{\mathbf{3}}$   $uu$  pair, which spin is compatible with Fermi statistics?

- A.  $j_d = 0$  only
- B.  $j_d = 1$  only
- C. Both  $j_d = 0$  and  $j_d = 1$
- D. Neither



# Hyperfine splitting

[Godfrey-Isgur, PRD 32 \(1985\) 189](#)

[PDG: Quark model \(2025\)](#)

$$\mathbf{s}_1 \cdot \mathbf{s}_2 = \frac{1}{2}[S(S+1) - s_1(s_1+1) - s_2(s_2+1)]$$

For two spin- $\frac{1}{2}$  constituents:

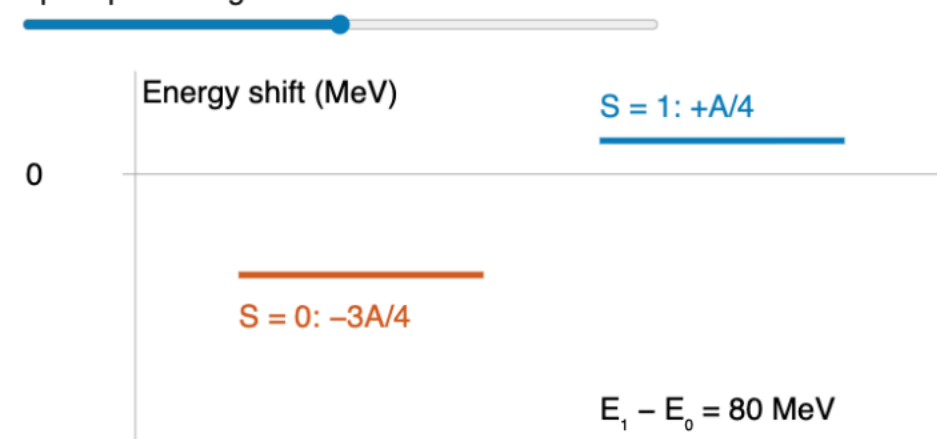
$$\langle \mathbf{s}_1 \cdot \mathbf{s}_2 \rangle = \begin{cases} -3/4 & S = 0, \\ +1/4 & S = 1. \end{cases}$$

With a fixed spatial wavefunction,

$$\Delta E_S = A \langle \mathbf{s}_1 \cdot \mathbf{s}_2 \rangle.$$

Vary  $A$ : the singlet and triplet separate while the spin-weighted centroid stays fixed.

Spin-spin strength  $A$ : 80 MeV



Fixed spatial wavefunction · first-order shifts · centroid  $(E_0 + 3E_1)/4 = 0$

Exact spin algebra; illustrative first-order model. Full GIModel spectra follow.

# Quark Model calculation

[Godfrey–Isgur, PRD 32 \(1985\) 189](#)

[GIModel.jl documentation](#)

[PDG: Quark model \(2025\)](#)

## One relativized $q\bar{q}$ Hamiltonian

**18 shared parameters** → light, heavy–light and heavy–quarkonium families.

Cornell potential: attraction + confinement

$$V_{\text{Cornell}}(r) = -\frac{4a_s}{3r} + br + c$$

Relativistic motion and spin interactions

$$H = H_{\text{central}} + H_{\text{SS}} + H_{\text{LS}} + H_{\text{tensor}}$$

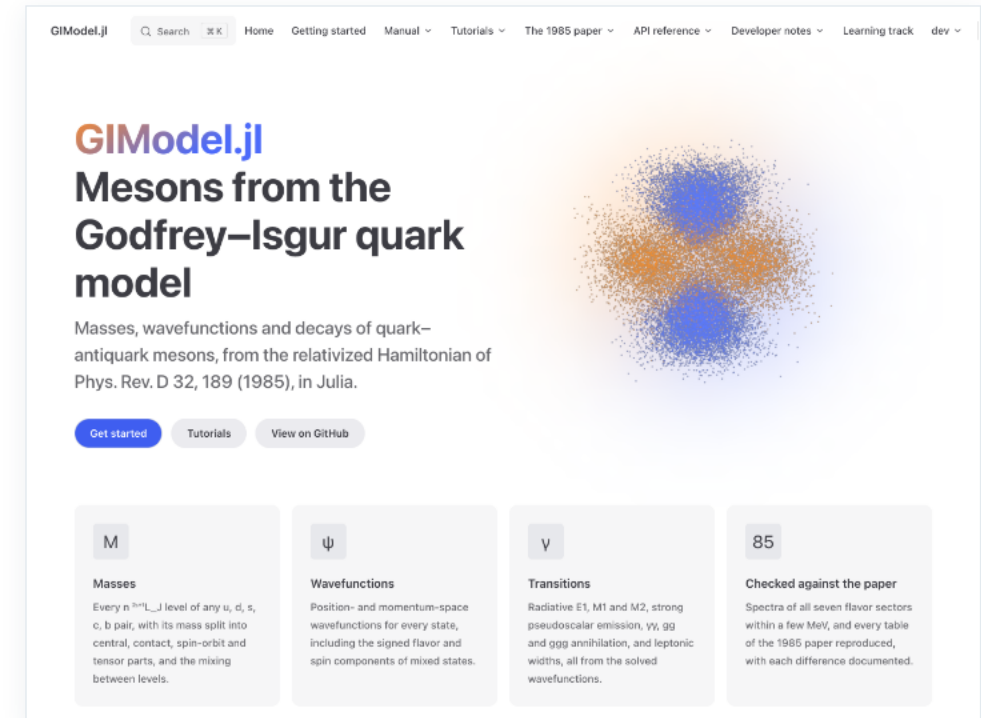
$$H_{\text{central}} = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} + \tilde{V}$$

GI adds **running coupling, smearing and relativistic factors**.

Radial:  $1S \rightarrow 2S \rightarrow 3S \rightarrow \dots$

Orbital:  $1S \rightarrow 1P \rightarrow 1D \rightarrow 1F \rightarrow \dots$

18 inputs for the ordinary spectrum; isoscalar annihilation mixing requires additional parameters.



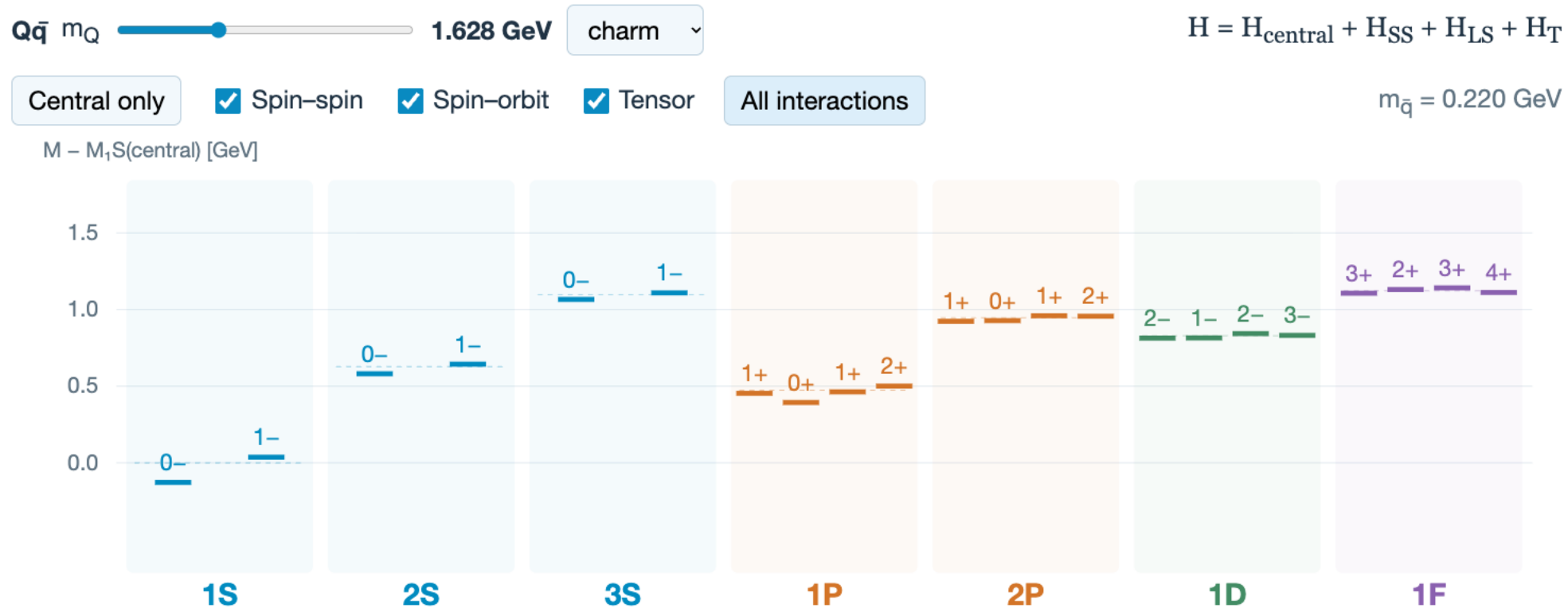
GIModel.jl · masses, wavefunctions and transitions

# Building a meson family

[Godfrey-Isgur, PRD 32 \(1985\) 189](#)

[GIModel.jl: numerical implementation](#)

[PDG: Quark model \(2025\)](#)



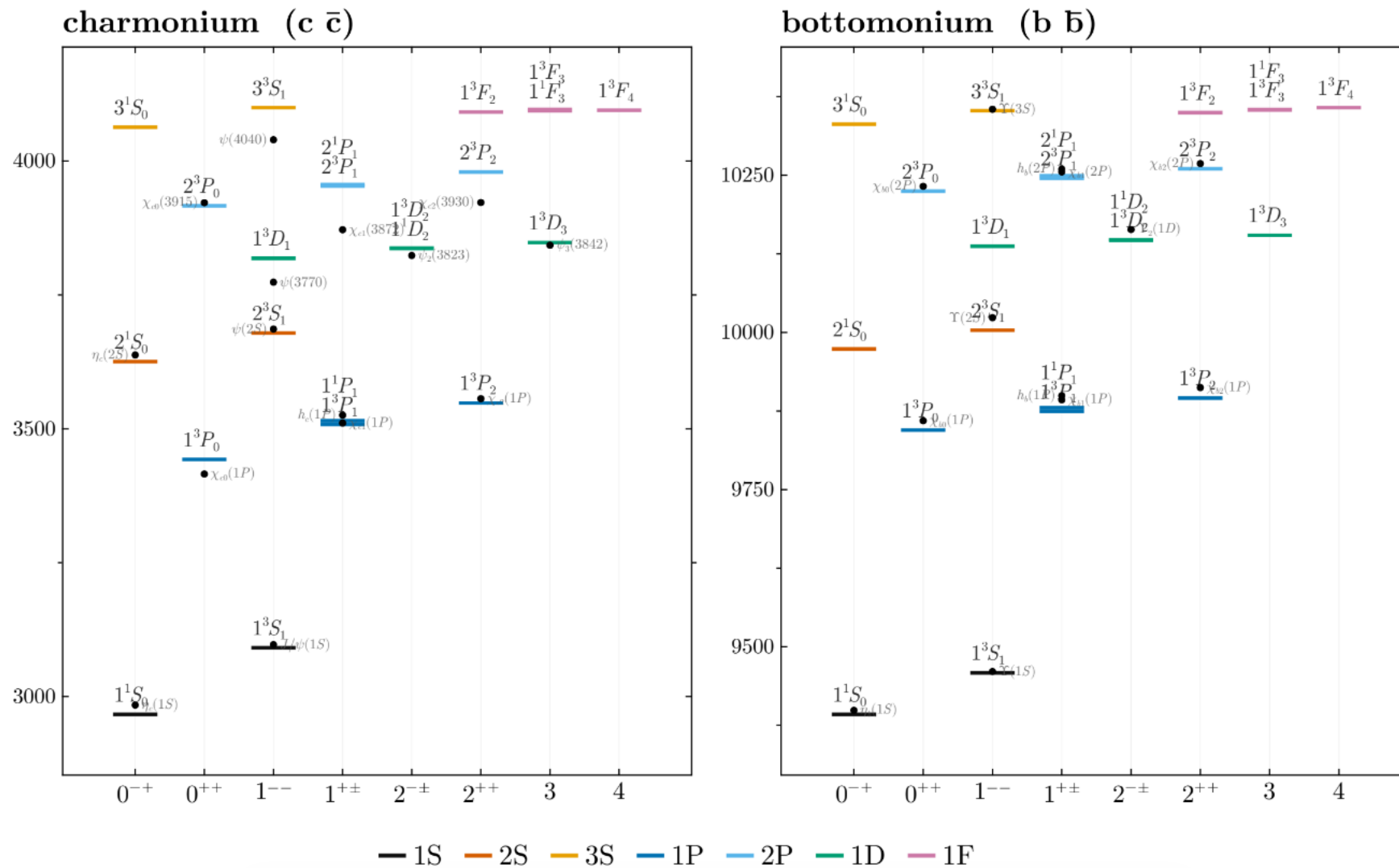
**7 radial-orbital levels → 22 meson states, for just one flavour pair**

GIModel numerical eigenvalues · dashed: central-only levels · labels:  $J^P$  · hover: mass and spectroscopic label. Constituent-mass scan; no interpolation. For unequal masses, spin-orbit mixes the two  $J = L$  states. Model states, not observed-particle assignments.

# Quarkonium spectra

Godfrey-Isgur, PRD 32 (1985) 189

GIModel.jl



# Observation of hadrons

C · Production, discoveries and the experimental spectrum

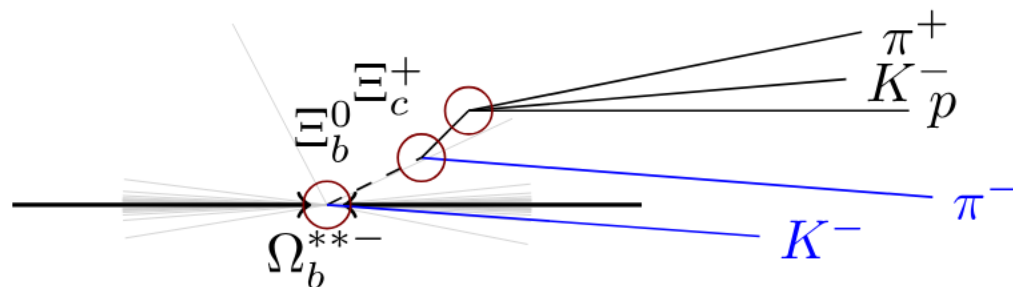
# How we search for hadrons

[TikZ Hadron Gallery · collision diagrams](#)

[LHCb: excited  \$\Omega\_b\$ , 2001.00851](#)

[LHCb:  \$B \rightarrow DDK\$ , 2009.00026](#)

## Direct (prompt) production



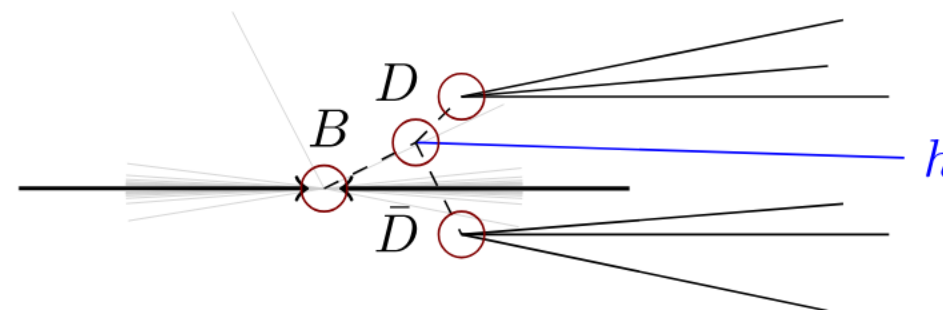
$$pp \rightarrow \Omega_b^{*-} + X, \quad \Omega_b^{*-} \rightarrow \Xi_b^0 K^-.$$

Reconstruct the long-lived  $\Xi_b^0$  and combine it with a **kaon from the collision vertex**.

Search for peaks in  $m(\Xi_b^0 K^-)$ .

The resonance is prompt; its weakly decaying daughters travel before decaying.

## Production in decays



$$B^+ \rightarrow D^+ D^- K^+, \quad T_{cs} \rightarrow D^- K^+.$$

Reconstruct a **displaced B decay** and inspect the invariant masses of its daughter pairs.

The  $D^- K^+$  channel reveals the  $T_{cs}$  (2900) structures.

Weak c-hadron decays similarly probe lighter hadron resonances.

# Observed hadrons: the experimental inventory

[PDG: Review of Particle Physics](#)  
[LHCb-FIGURE-2026-010](#)



The particle physicist's reference book

Photo: PDG · 2024 edition

## From a spectrum to a catalogue

**Masses, widths, quantum numbers and decay modes** bring experimental results together.

# 756

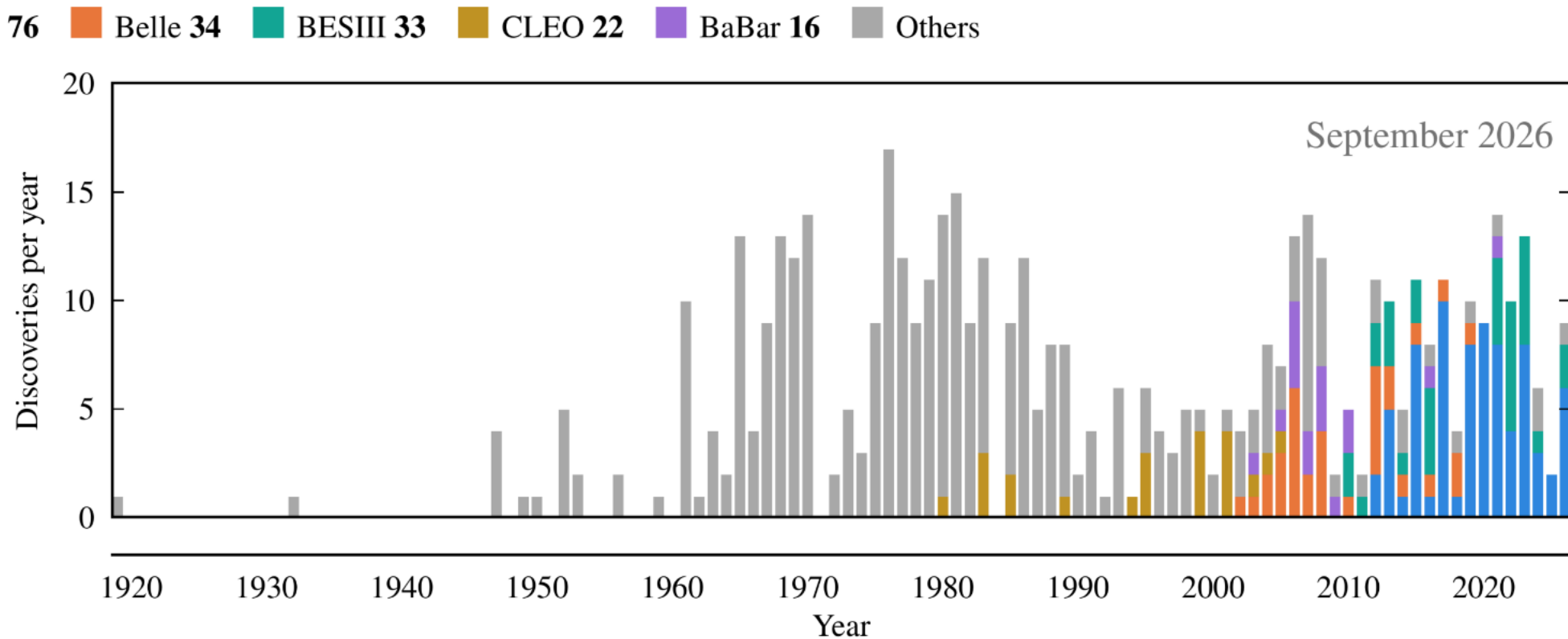
## hadron entries

Including “**Further States**”: some entries are **less established** or **controversial**.

Not all entries have the same level of experimental support.

# A century of hadron discoveries

LHCb-FIGURE-2026-010 · September 2026



New beams and detectors keep revealing new states.

# Where the discoveries happened

[LHCb-FIGURE-2026-010](#)



**Bars span each machine's first to last discovery—not its operating lifetime.**

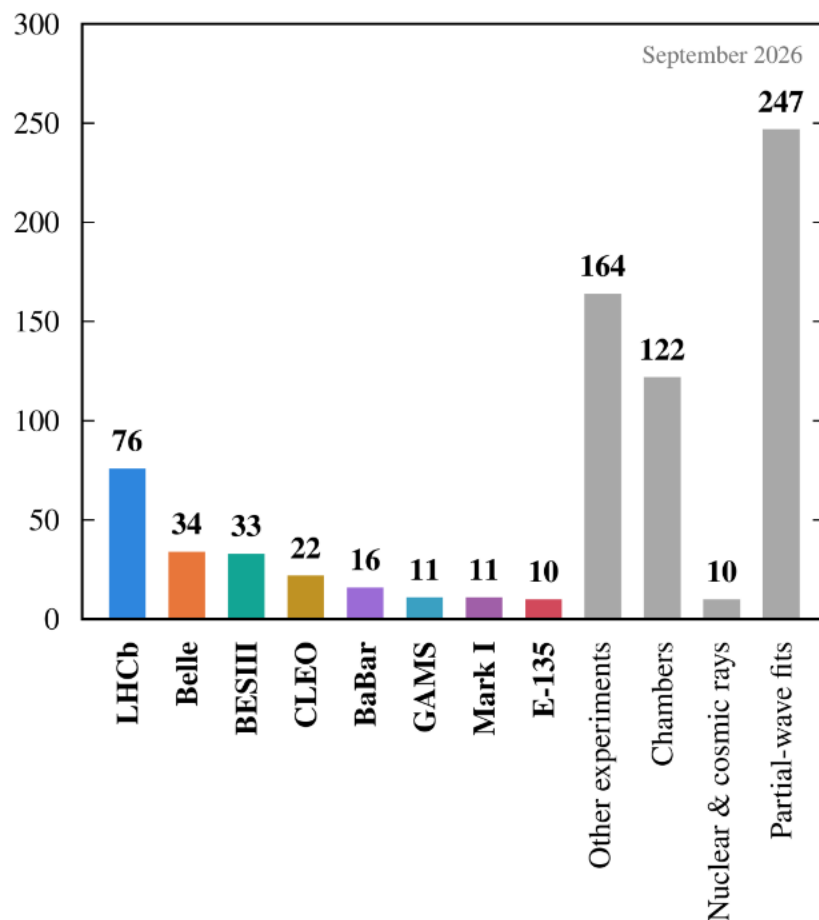
# Discoveries also come from amplitude analyses

[LHCb-FIGURE-2026-010 · September 2026](#)

[LHCb:  \$J/\psi\phi\$  amplitude analysis \(2016\)](#)

[LHCb: pentaquark amplitude analysis \(2015\)](#)

All hadrons



## Not every state is an isolated peak

**Overlapping resonances** can require a partial-wave analysis to separate their contributions.

**LHCb discoveries also use amplitude analyses**—for example, the  $J/\psi\phi$  states and hidden-charm pentaquarks.

The bars show **discovery attribution**, not the analysis method. “Partial-wave fits” is not the total number of states found using such fits.

# Heavy-flavour baryons: organise the families

[PDG: Quark model](#)

[PDG: Baryon summary tables](#)

A heavy quark plus two light quarks:  $Qqq$ ,  $Q = c$  or  $b$

| Light-quark pair         | Charm families           | Bottom families          |
|--------------------------|--------------------------|--------------------------|
| $ud$ · isospin 0         | $\Lambda_c$              | $\Lambda_b$              |
| $uu, ud, dd$ · isospin 1 | $\Sigma_c, \Sigma_c^*$   | $\Sigma_b, \Sigma_b^*$   |
| $us$ or $ds$             | $\Xi_c, \Xi'_c, \Xi_c^*$ | $\Xi_b, \Xi'_b, \Xi_b^*$ |
| $ss$                     | $\Omega_c, \Omega_c^*$   | $\Omega_b, \Omega_b^*$   |

**Each flavour family can support a ladder of orbital and radial excitations.**

Family labels, not a checklist of observed states. Here \* denotes the spin-3/2 ground-state partner.

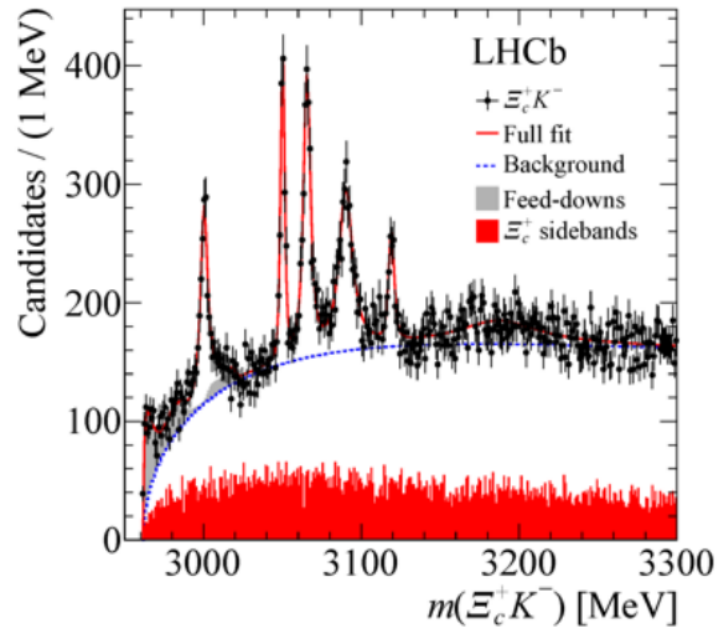
# Resolving excited $\Omega$ baryons

[LHCb:  \$\Omega\_c\$ , PRL 118 \(2017\) 182001](#)

[LHCb:  \$\Omega\_b\$ , PRL 124 \(2020\) 222001](#)

Charm · css · 2017

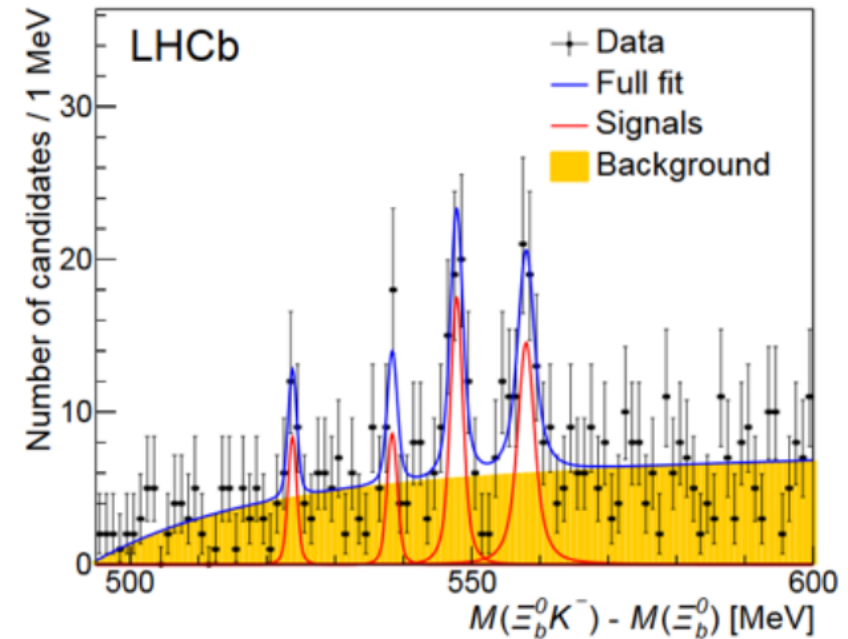
Reconstruct  $\Xi_c^+ K^-$ : five narrow peaks.



$\Xi_c^+ \rightarrow p K^- \pi^+$

Bottom · bss · 2020

Reconstruct  $\Xi_b^0 K^-$ : four narrow structures.



$\Xi_b^0 \rightarrow \Xi_c^+ \pi^-$ , followed by  $\Xi_c^+ \rightarrow p K^- \pi^+$

# Two heavy quarks: the $\Xi_{cc}^{++}$ discovery

LHCb, PRL 119 (2017) 112001 · Fig. 2

CERN · 6 July 2017

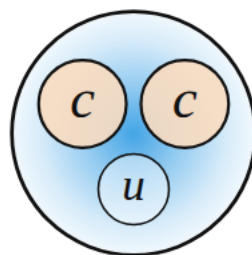
## Reconstruct a weak-decay chain

$$\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+, \quad \Lambda_c^+ \rightarrow p K^- \pi^+$$

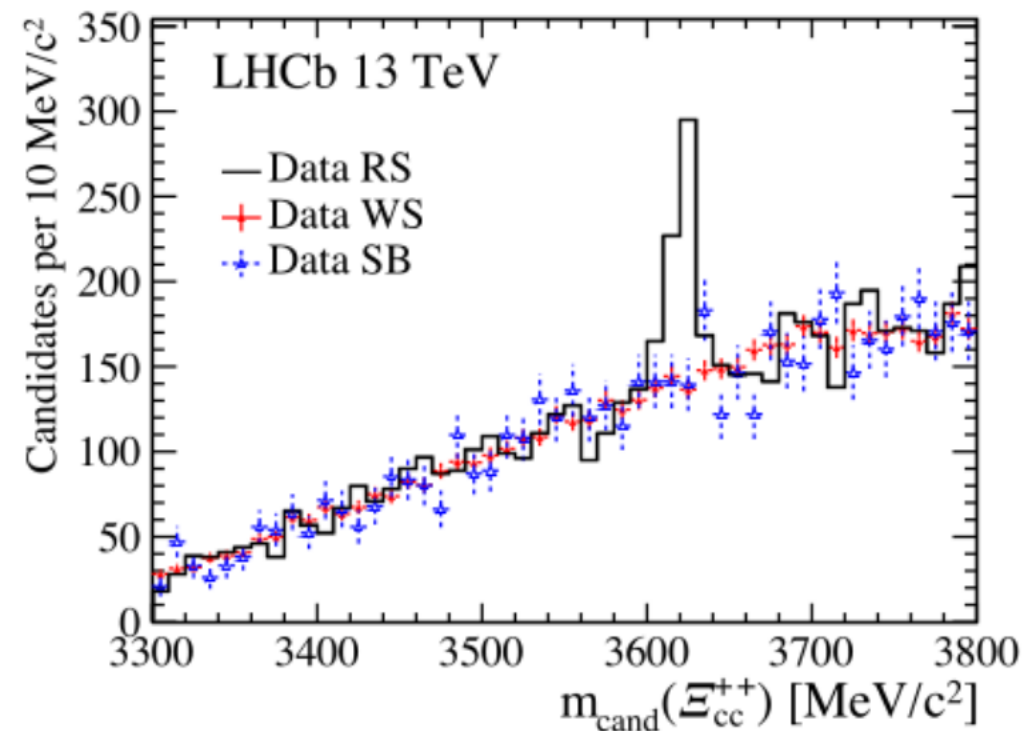
**Quark content: ccu.**

A narrow signal near **3621 MeV**.

Check alternative charge combinations and  $\Lambda_c$  sidebands: the peak is absent.

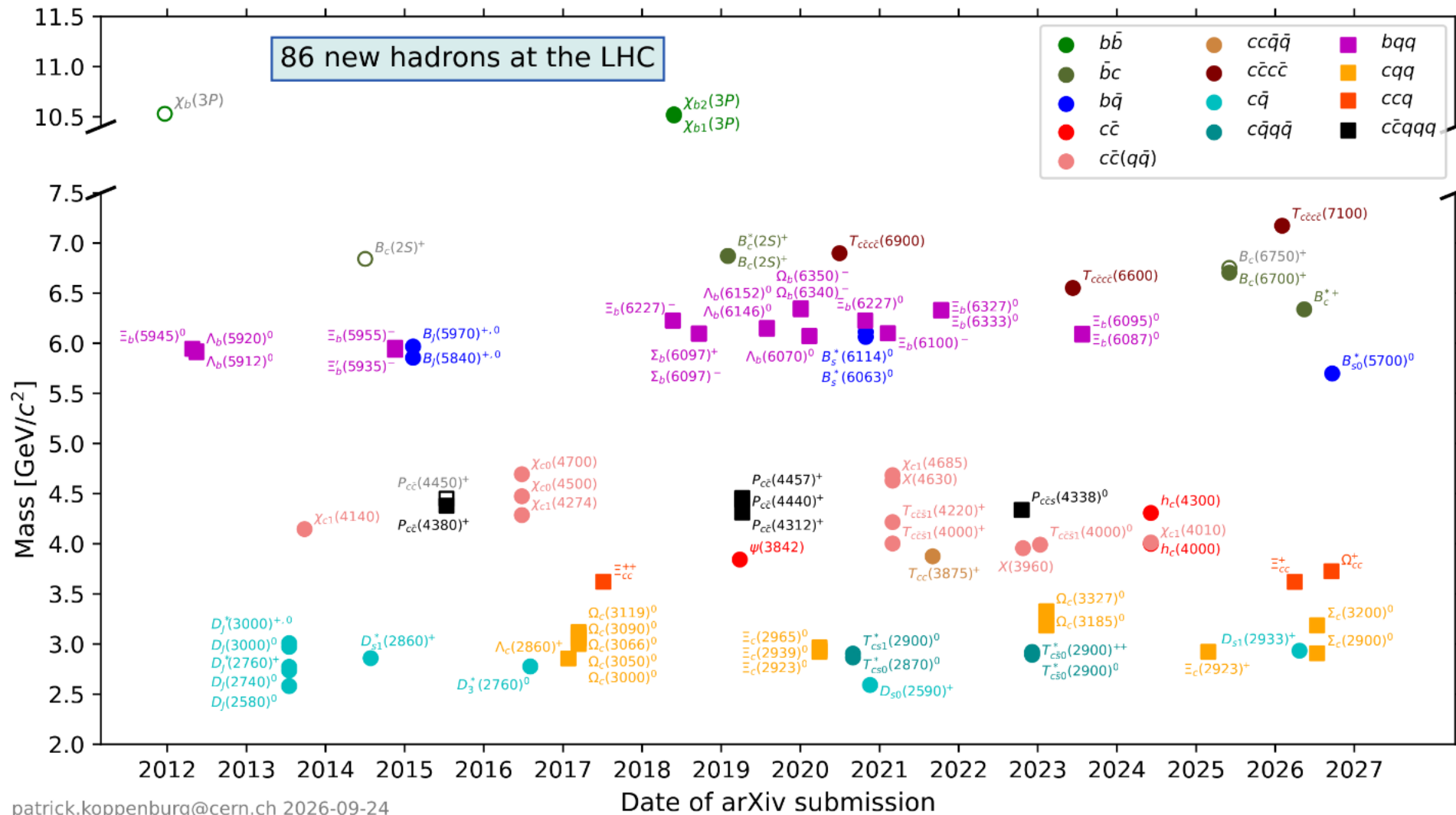


A historical milestone in 2017: a baryon containing two charm quarks.



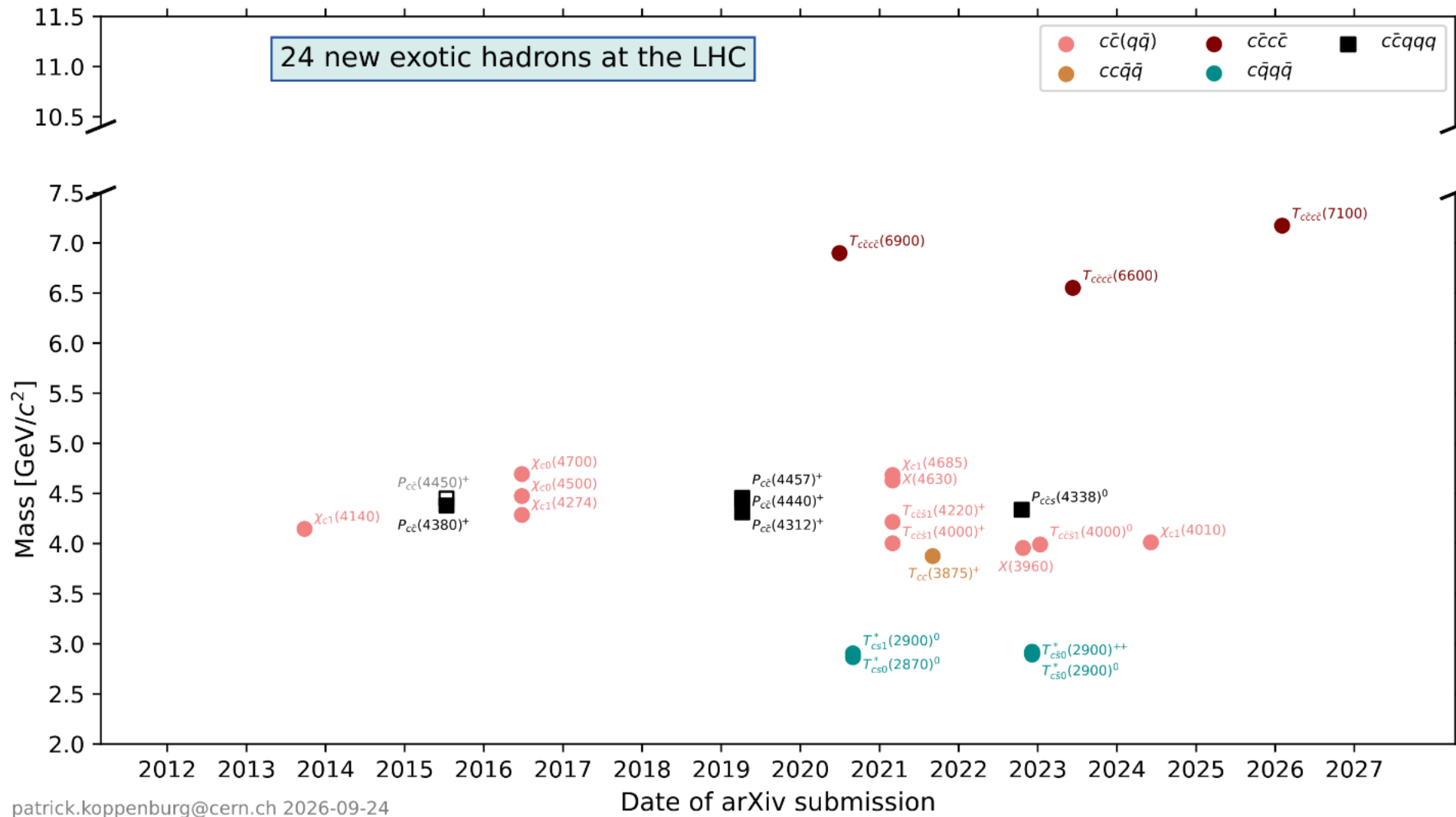
# New hadrons at the LHC

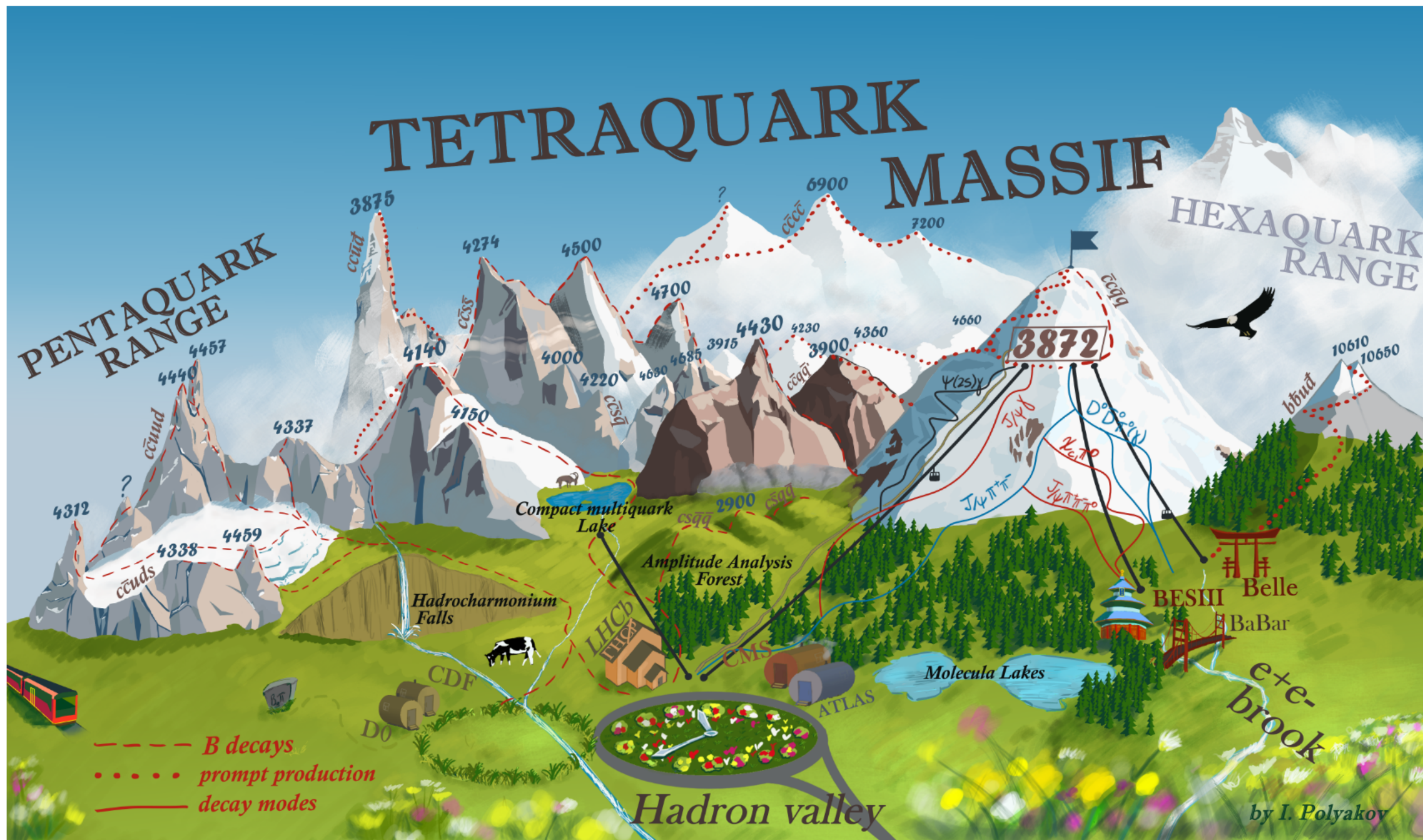
P. Koppenburg · 24 September 2026



# Exotic hadrons at the LHC

P. Koppenburg · 24 September 2026





# Break

# Decays and the hadronic continuum

D · Resonances, thresholds and exotic hadrons

# QCD states as resonances

Most excited hadrons can decay through the strong interaction:

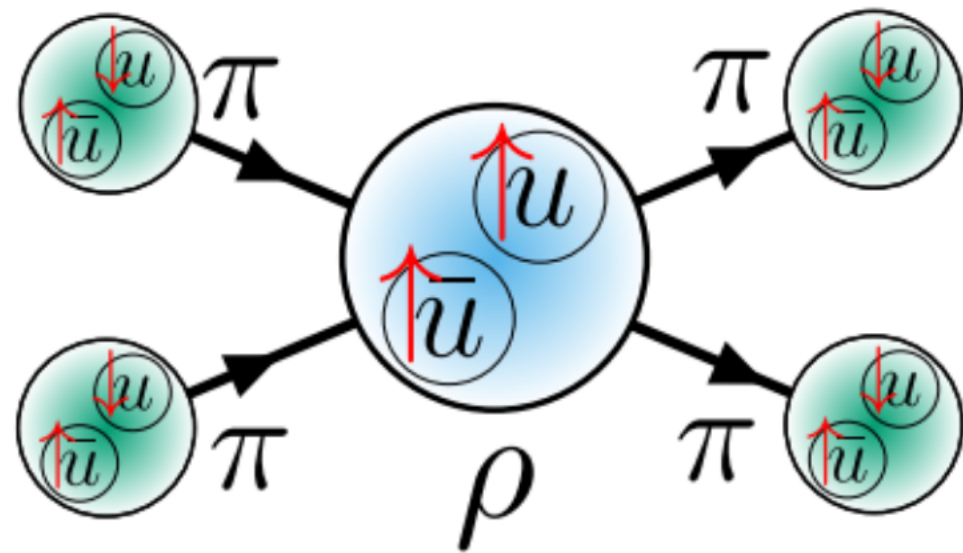
$$P_0 \rightarrow P_1 + P_2.$$

The same quantum numbers belong to a **two-hadron continuum**: the pair can carry any relative momentum.

A short-lived excitation appears in the interaction of these hadrons — for example,  $\pi\pi \rightarrow \rho \rightarrow \pi\pi$ .

For an isolated, narrow resonance,

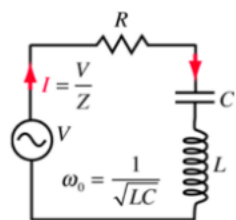
$$\tau \approx \frac{\hbar}{\Gamma}.$$



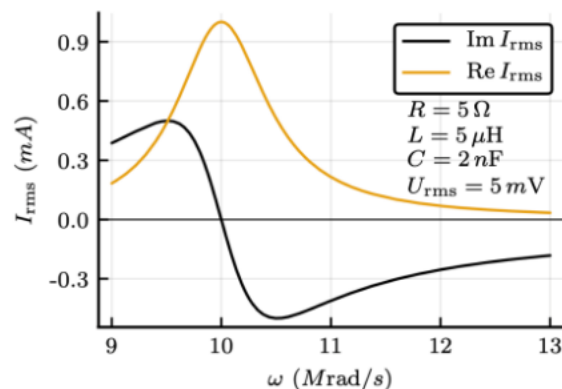
A quark-model level becomes a resonance when coupled to open decay channels.

# Resonances

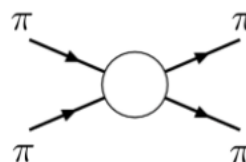
## Electrical resonance



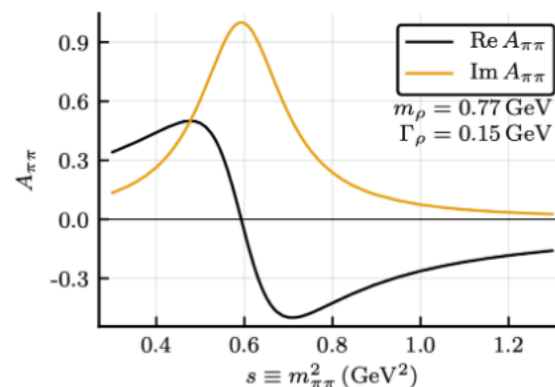
$$I_{\text{rms}} = \frac{U}{R + iL\omega - \frac{i}{C\omega}}$$



## Hadronic resonance



$$A_{\pi\pi} = \frac{m\Gamma}{m^2 - s - im\Gamma}$$



An oscillating circuit and a hadronic system share the same language:

**Enhanced response** near a characteristic energy.

**Phase motion** as the energy passes through it.

**A pole** of the complex response function.

Interference can turn a resonance peak into a dip.

# Hadronic amplitude

## The experiment measures an intensity

$$\frac{dN}{d\Phi} \propto \epsilon(\Phi) \sum_{\text{unobserved spins}} |\mathcal{A}(\Phi)|^2.$$

$\Phi$ : final-state kinematics;  $\epsilon$ : detection efficiency.

Amplitudes for the **same final state interfere** before squaring.

## A simple resonance model

$$\mathcal{A}(s) = \frac{N(s)}{m_0^2 - s - i \sum_i g_i^2 \rho_i(s)}, \quad s = E^2.$$

The production process changes  $N(s)$ ; the resonance pole is shared between reactions.

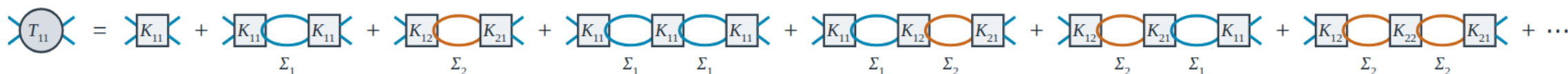
## What goes into the width?

**Far from relevant thresholds:** a constant  $m_0 \Gamma$  can approximate the imaginary term.

**One important channel:** retain  $g^2 \rho(s)$ , with two-body phase space  $\rho(s) = 2k/\sqrt{s}$ .

**Several channels:** sum their contributions,  $\sum_i g_i^2 \rho_i(s)$ .

Near a threshold, phase space varies rapidly. A constant-width peak can be misleading.



# The amplitude in the complex plane

## What is A?

A **transition amplitude (matrix element)** for chosen initial and final channels, projected onto a **fixed partial wave**.

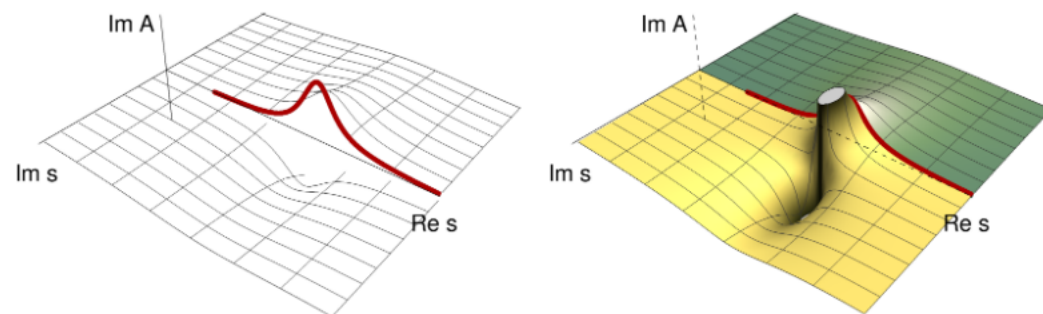
$$A(s) \equiv A_{ba}^J(s).$$

The angular dependence has been projected out; channel and spin labels are suppressed.

## What is s?

$$s = (p_1 + p_2)^2 = E_{\text{cm}}^2.$$

The **invariant mass squared** of the system whose resonance we study.



**Analytic continuation:** extend the same analytic amplitude away from real  $s$ , respecting its singularities and branch choices.

$$s \in \mathbb{R} \quad \rightarrow \quad s = x + iy \in \mathbb{C}.$$

$A(s)$  can already be complex for **real**  $s$ .

# Guiding principles for amplitudes

## I Analyticity ← Causality

Causal dynamics constrains how the amplitude extends into complex energy.

---

## II Unitarity ← Probability conservation

Probability is conserved when all accessible final channels are included.

---

## III Crossing symmetry

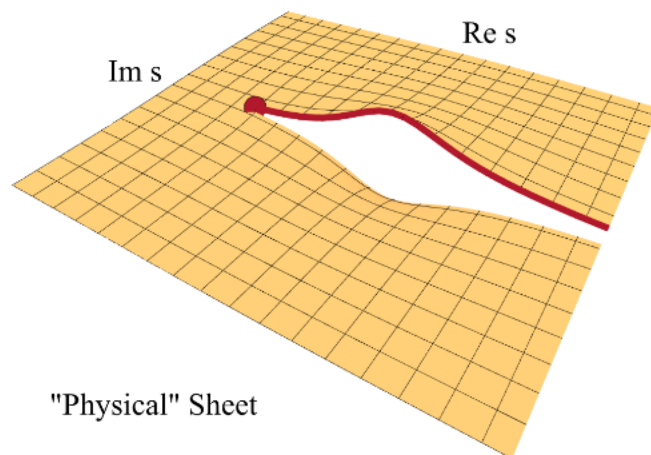
Related reactions are described by continuations of the same underlying amplitude.

---

## Consequences: analytic structure

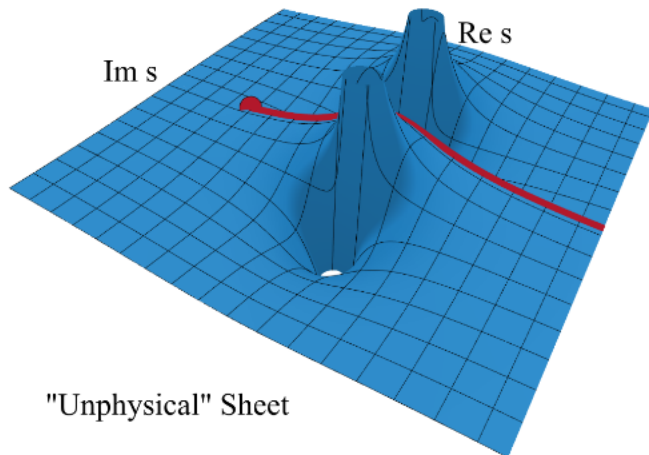
- **Hadronic resonances** → **poles** on unphysical sheets.
- **Decay-channel thresholds** → **branch points**, joining a non-trivial set of **Riemann sheets**.

# First and second Riemann sheets



## Physical sheet

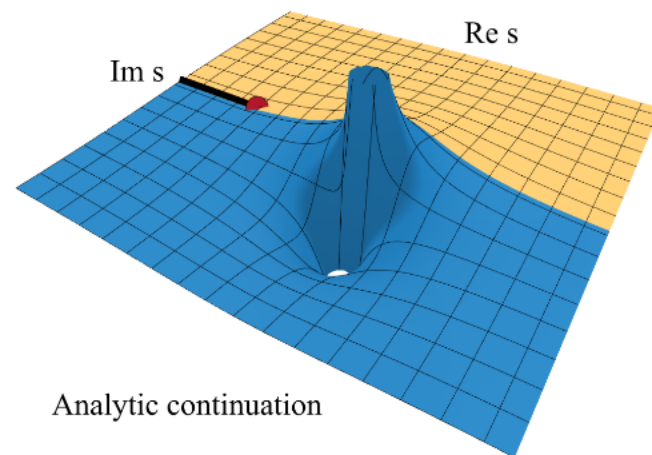
The red curve follows the physical real-energy axis.



## Second sheet

Resonance poles influence the amplitude on the physical axis.

$$k(s) \propto \sqrt{s - s_{\text{th}}} \quad \text{near threshold,} \quad k_{\text{II}} = -k_{\text{I}}.$$



## Analytic continuation

The upper physical sheet joins the lower second sheet across the cut.

A threshold is a **branch point**: the two signs of the square root lead to two sheets.

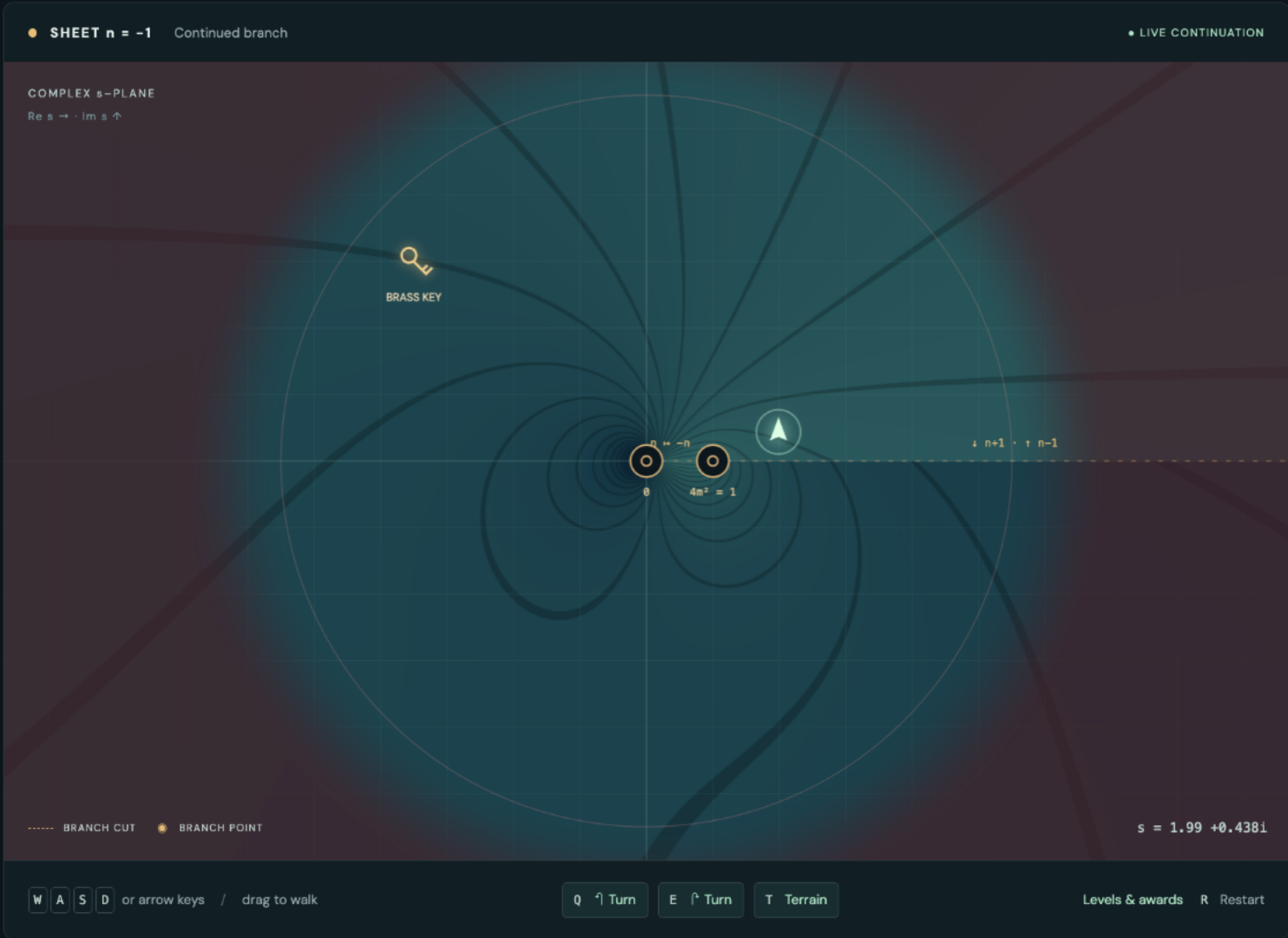
EXPEDITION 06

# One plane. A hidden threshold.

Chew–Mandelstam ·  $m = \frac{1}{2}$  · The origin is regular on the physical sheet.

$$f(s) = \Sigma I(s) + 2in\rho_p(s)$$

LEVEL 06 · INFINITELY MANY SHEETS



## YOUR EXPEDITION

### Threshold

The same coordinates can belong to different worlds. Find a way between them.

Find the brass key ○  
Sheet  $n = -1$  ·  $s = -3.500 + 3.000i$

Open the hidden chest ○  
Sheet  $II$  ( $n = 1$ ) ·  $s = 3.500 - 3.000i$

STABILITY 100%

Beyond  $|s| = 5.5$ , infinity pulls you away.

## OBSERVATION

Perspective

Top Spot Player

Terrain height / colour

Im  $f$  — imaginary part

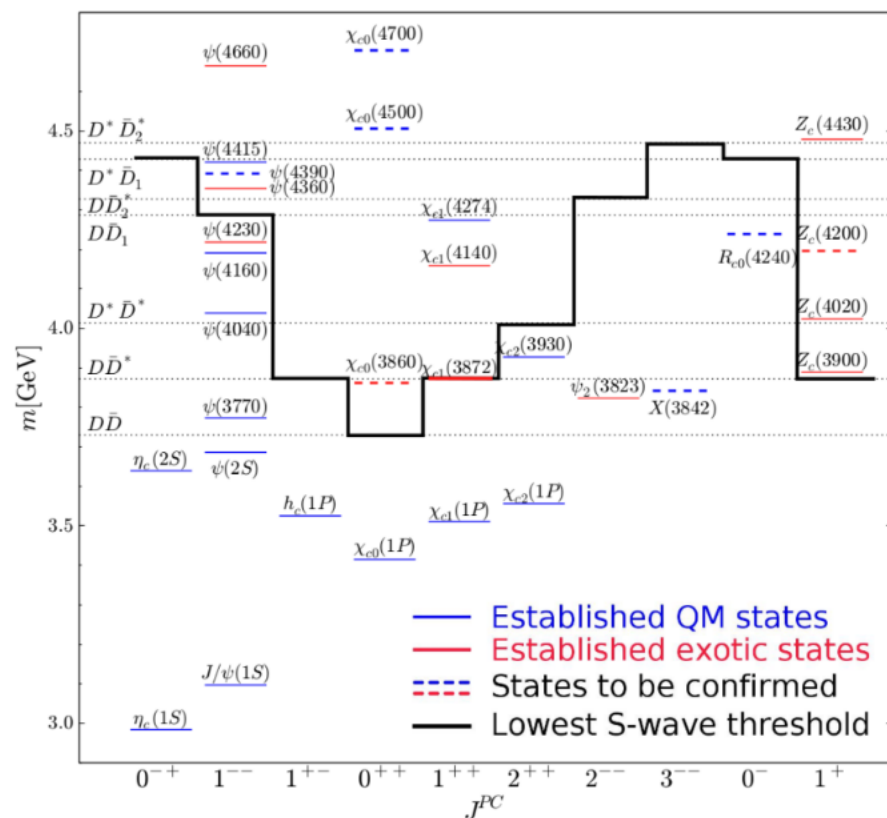
−4 Im  $f$  +4

☒ Easy mode — show key & chest

Reveal distant markers on viewed sheets; in Player view, only in the light. Collect them on the required sheet.

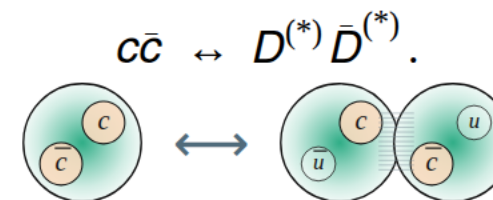
The entire terrain changes when you cross the cut. Keys and chests appear nearby; Easy mode

# Quark-model states and thresholds



Charmonium and the lowest S-wave thresholds · historical overview

A quark-model level is **not isolated** from hadron pairs with the same quantum numbers.



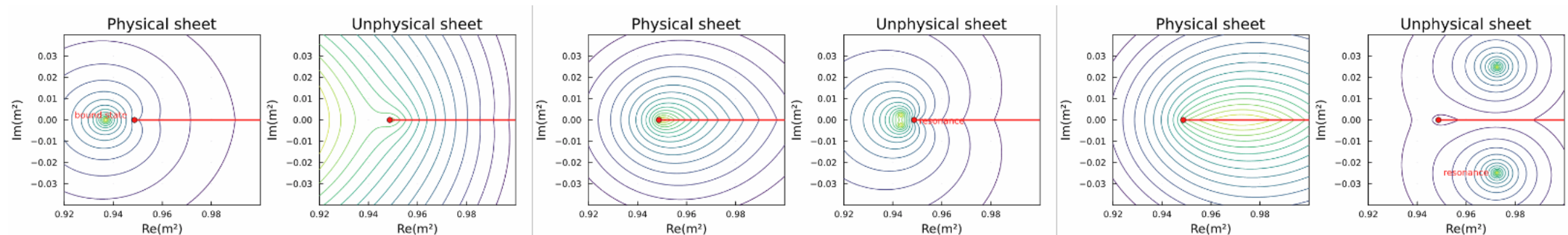
The coupling shifts levels and, above an allowed threshold, opens decays.

For a bound state, schematically,

$$|\psi\rangle = a|Q\bar{Q}\rangle + \sum_i b_i |H_{1i} H_{2i}\rangle.$$

The hadron-pair part is the **molecular component**.

# Why a molecule can be a good model



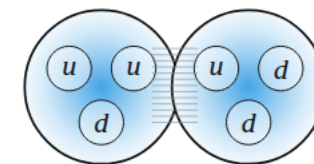
**Bound  $\rightarrow$  virtual  $\rightarrow$  resonance:** a pole can move continuously as the interaction changes.

These are different pole locations, not three unrelated kinds of constituent.

A shallow S-wave bound state has a long-distance tail:

$$\psi(r) \propto \frac{e^{-\kappa r}}{r}, \quad \kappa = \sqrt{2\mu B}, \quad B = E_{\text{th}} - E_{\text{bound}}.$$

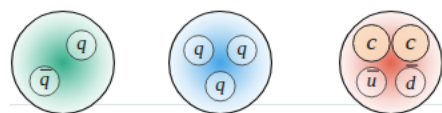
When  $1/\kappa$  exceeds the interaction range, a hadron-pair description becomes especially useful.



# Summary and discussion

## Hadron families

**10 + 24 + X**



**Conventional hadrons probe QCD in different regimes**—from light-quark dynamics to heavy-quark systems.

**Exotic hadrons** expand the possibilities for how QCD binds matter.

## Experimental discoveries

**A golden age of hadron spectroscopy: the past 15 years.**

**Belle / Belle II + BESIII + LHC:** new forms of hadronic matter, a richer understanding of what QCD allows—and **many puzzles still open.**

## Hadrons as resonances

**S-matrix-driven physics:** understand hadrons through their scattering and decay amplitudes.

Intricate mathematics of **cuts, poles and Riemann sheets**—with ideas reaching far beyond hadron physics.

Hadronic S-matrix → **Veneziano amplitude** → **string theory**