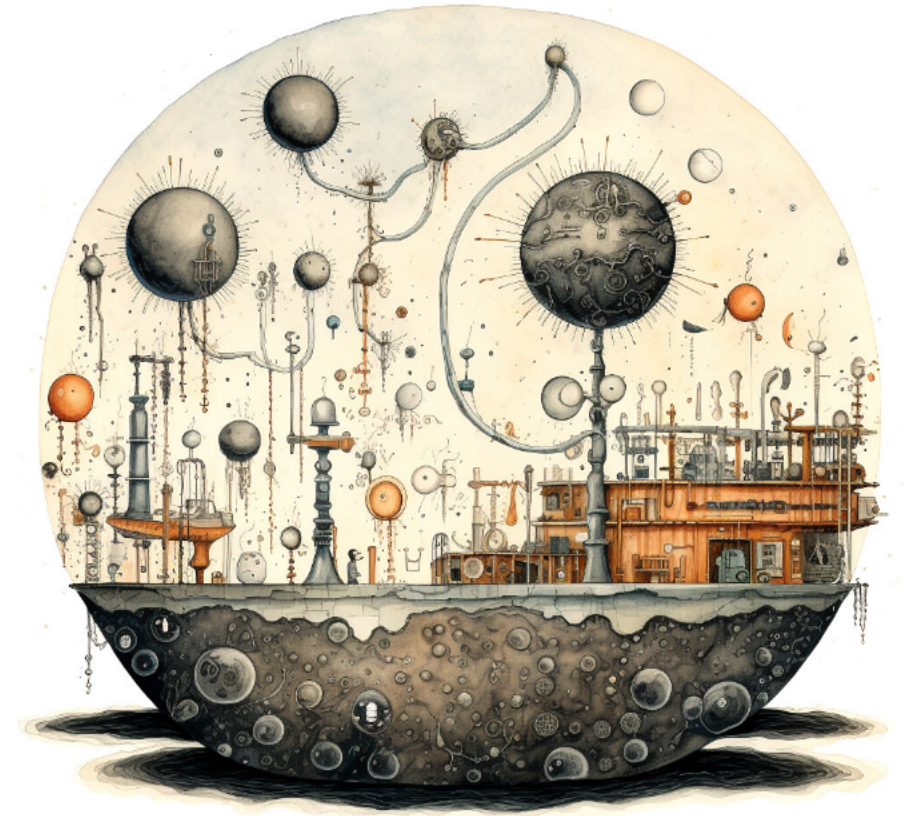


Hadron Spectroscopy

Amplitude analysis: how we find hadrons in decays



Mikhail Mikhasenko · Lecture 2
AEPSHEP 2026 · Taipei

Lecture 2 roadmap

- A** **The puzzle** — a peak in J/ψ p
- B** **Kinematics** — phase space, the Dalitz plot, bands and reflections
- C** **Spin** — rotations, decay chains, the full amplitude
- D** **Mass dependence** — line shapes and form factors
- E** **The full model** — $\Lambda_b \rightarrow J/\psi$ p K^- and the pentaquarks
- F** **Near threshold** — T_{cc}^+

The puzzle

A peak is not yet a particle

A narrow peak in $J/\psi p$

LHCb · PRL 115, 072001 (2015)

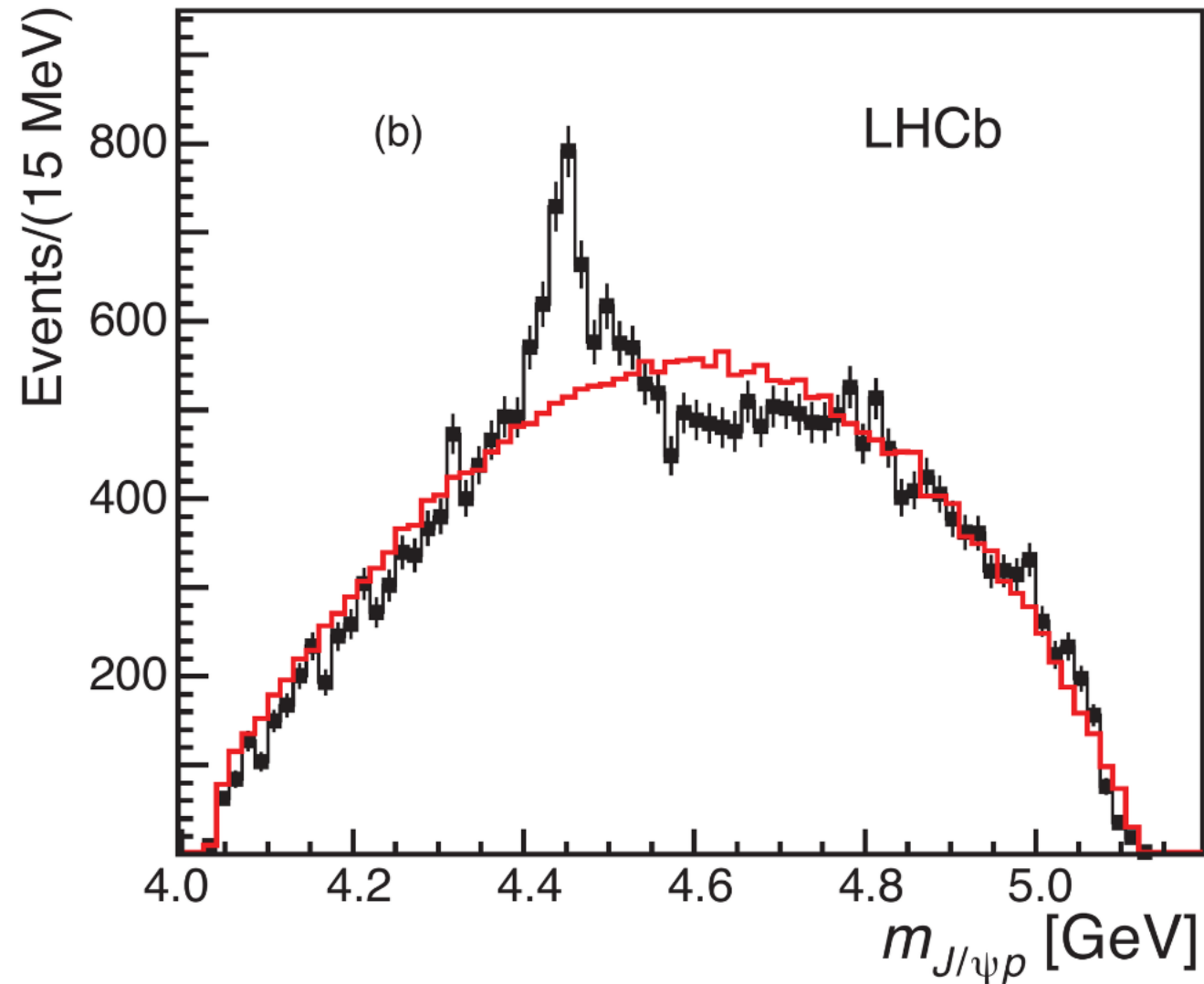
LHCb 2015: about 26 000

$\Lambda_b^0 \rightarrow J/\psi p K^-$ decays, almost no background.

The $J/\psi p$ mass shows a **narrow peak** near 4.45 GeV. Phase space (red) has none.

A $J/\psi p$ resonance would contain $c\bar{c}uud$:
at least five quarks.

Is it a particle? How would we know?



Reconstructing $\Lambda_b \rightarrow J/\psi \, p \, K^-$

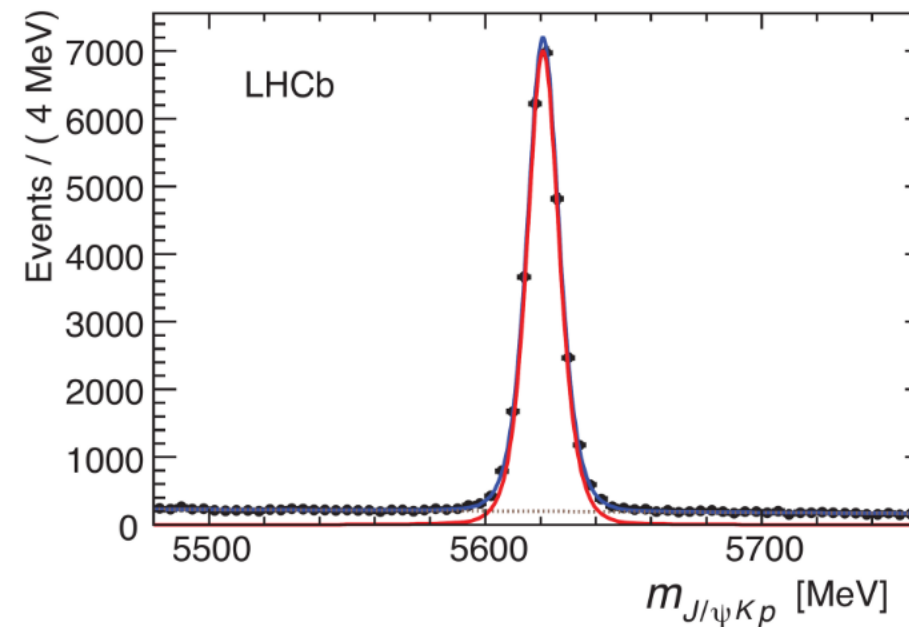
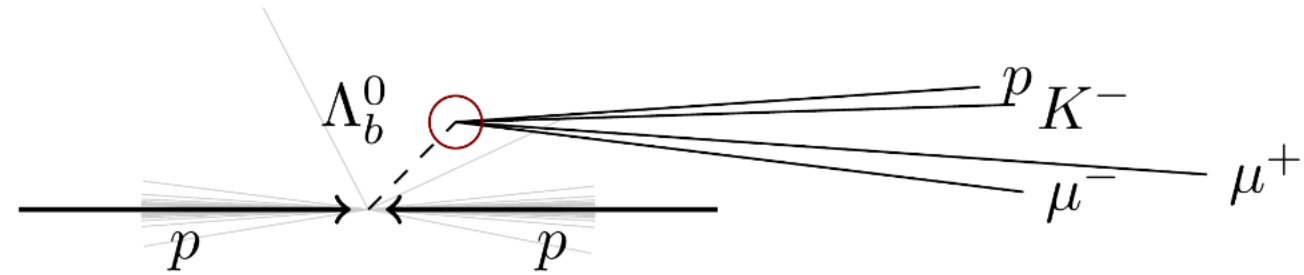
[LHCb · PRL 115, 072001 \(2015\)](#)

[MM · collision sketches](#)

A Λ_b is produced in a pp collision and flies a few millimetres before it decays weakly.

Four charged tracks — p , K^- , μ^+ , μ^- — come from one displaced vertex. The muon pair has the J/ψ mass.

Exclusive: every final-state particle is measured. Each event is a full set of four-momenta.



The Dalitz plot of the data

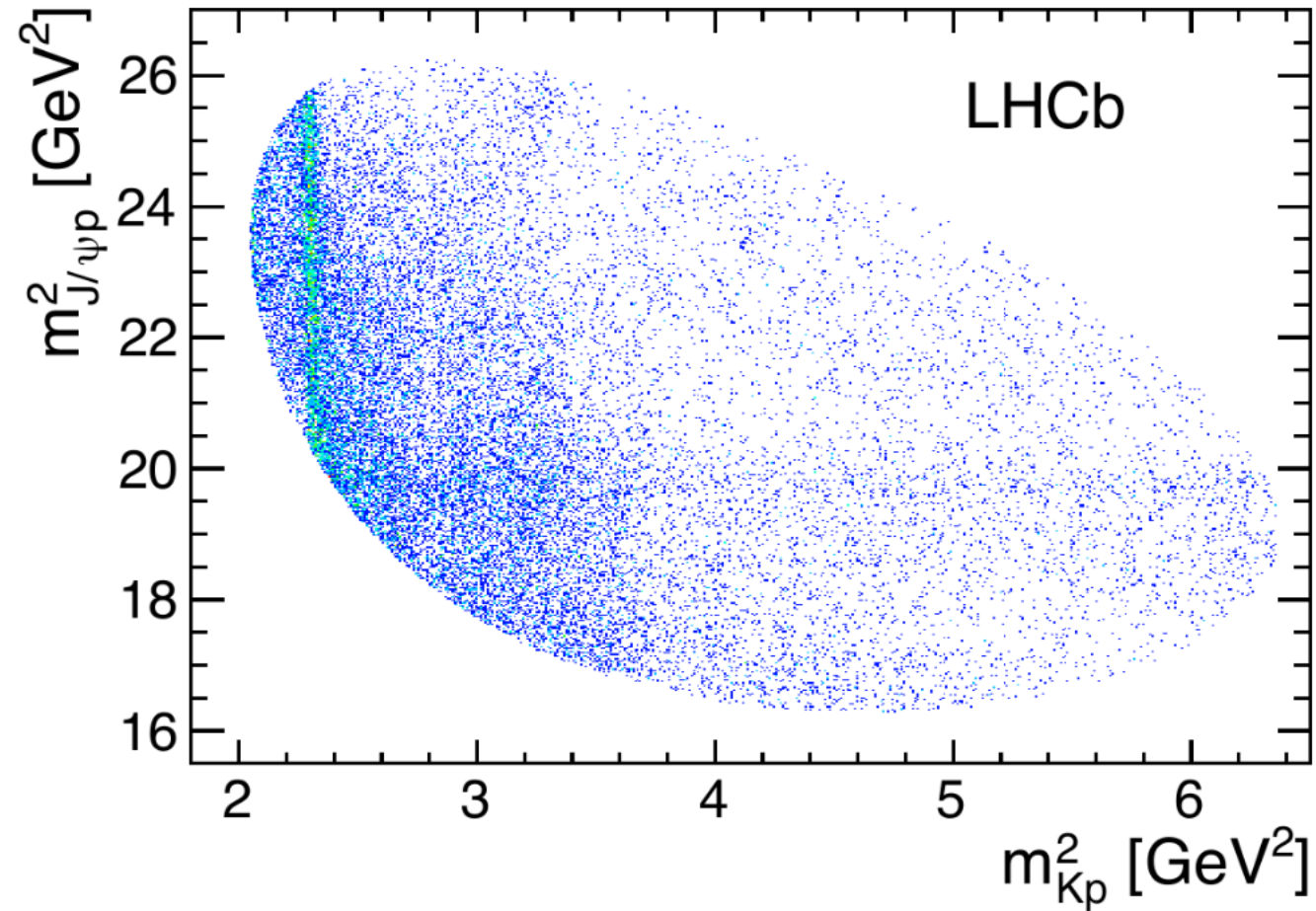
[LHCb · PRL 115, 072001 \(2015\)](#)

Each event is one point: two pair masses squared.

Vertical bands: $\Lambda^* \rightarrow pK^-$ resonances, strongest the $\Lambda(1520)$.

Horizontal band at $m^2(J/\psi p) \approx 19.8$ GeV^2 : the new structure.

The density is not uniform. Part of it is detector efficiency, the rest is physics.



Kinematics

What can vary in a three-body decay

From events to a probability

One event tells us little. Many events sample a **density** over the allowed momenta x .

$$dN \propto \epsilon(x) I(x) d\Phi(x)$$

Lecture 1: amplitudes for the same final state **interfere before squaring**.

Today: build $A(x)$ for $\Lambda b \rightarrow J/\psi p K^-$, then fit it to the events.

Three factors, three questions

$d\Phi(x)$ — **phase space**: what kinematics allows.

$I(x) = \sum_{\text{spins}} |\mathcal{A}(x)|^2$ — **dynamics**: what we want to learn.

$\epsilon(x)$ — **detector**: efficiency, known from simulation.

A structure in the data is physics only if it is in $I(x)$.

Phase space

$$d\Phi_n = \prod_{i=1}^n \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^{(4)}(P - \sum_{i=1}^n p_i)$$

Each particle

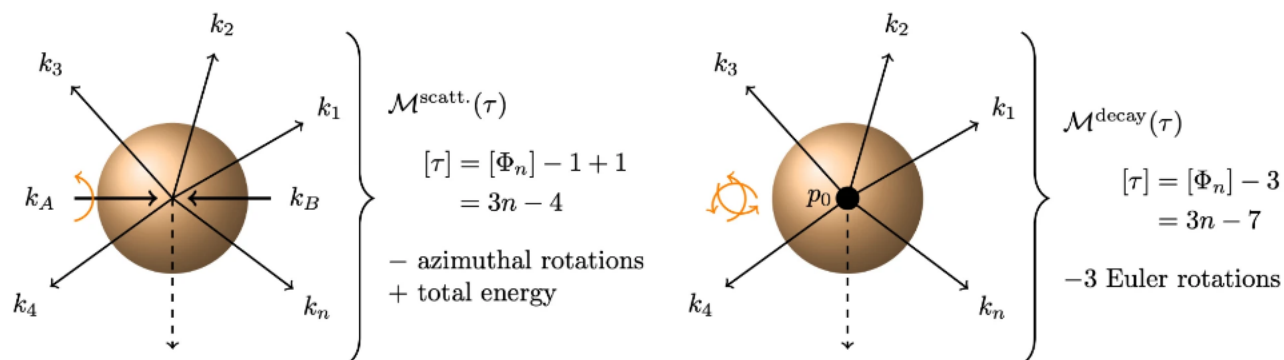
Three momentum components $\mathbf{p}_i$.
Dividing by $2E_i$ counts states the same way in every frame.

On the mass shell

$E_i = \sqrt{m_i^2 + |\mathbf{p}_i|^2}$: the energy is fixed by the mass, not a new number.

Conservation

Total energy and momentum equal the parent's: four constraints remove **4** numbers.



Two-body decay: one fixed momentum

In the parent rest frame the daughters fly back to back.

Energy conservation fixes the momentum:

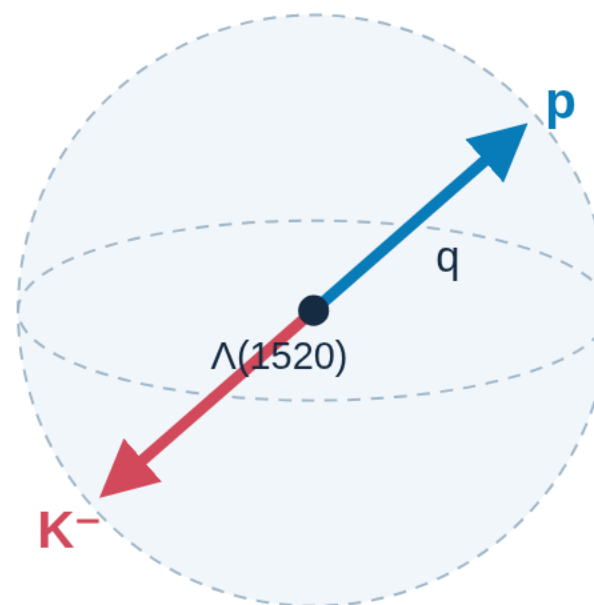
$$q = \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)}}{2M}$$

Only the direction is free:

$$d\Phi_2 = \frac{1}{8\pi} \frac{2q}{M} \frac{d\Omega}{4\pi}$$

$2q/M \rightarrow 1$ for massless daughters; $d\Omega/4\pi$: all directions equally likely.

$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. Derivation in backup.



$\Lambda(1520) \rightarrow p K^-$: $q = 0.243$ GeV, any direction

Recursive phase space formulation

[Byckling & Kajantie · Particle Kinematics \(1973\)](#)

[H. Jonas, DEMOS, Decay-Web-Page](#)

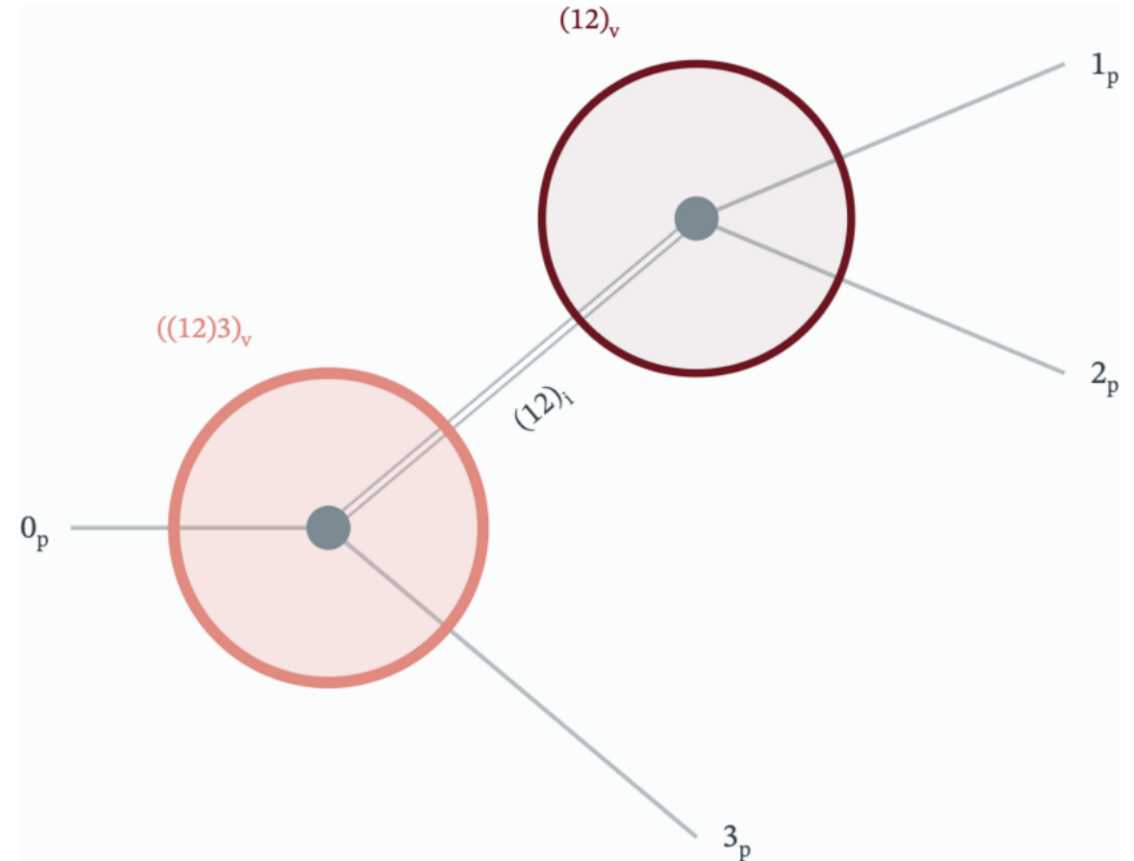
Group p and K^- : first $\Lambda_b \rightarrow (pK^-) J/\psi$, then $(pK^-) \rightarrow pK^-$.

$$d\Phi_3 = \frac{dm_{pK}^2}{2\pi} d\Phi_2 d\Phi_2$$

An **exact** identity for every event: no resonance is assumed. Resonances will live in the same two steps.

Integrate the three orientation angles:

$$d\Phi_3 \propto dm_{pK}^2 dm_{J/\psi p}^2$$



H. Jonas, DEMOS, Decay-Web-Page: the chain $0 \rightarrow (12) + 3$, then $(12) \rightarrow 1 + 2$.

Five numbers: shape and orientation

[R. H. Dalitz · Phil. Mag. 44, 1068 \(1953\)](#)

[JPAC · PRD 101, 034033 \(2020\)](#)

Three momenta: $3 \times 3 = 9$ numbers.

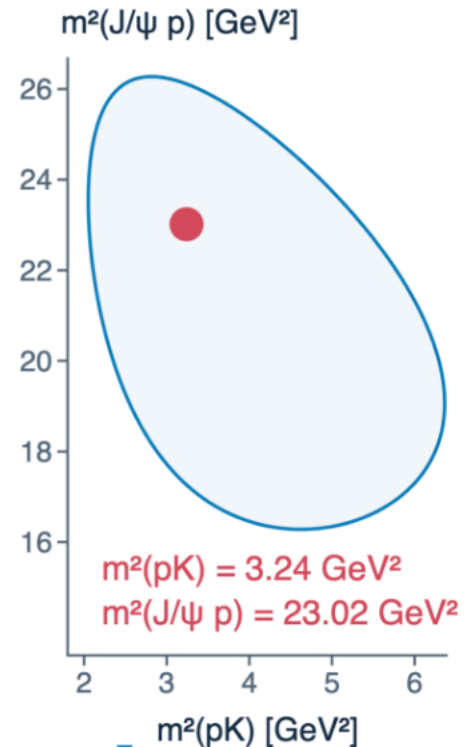
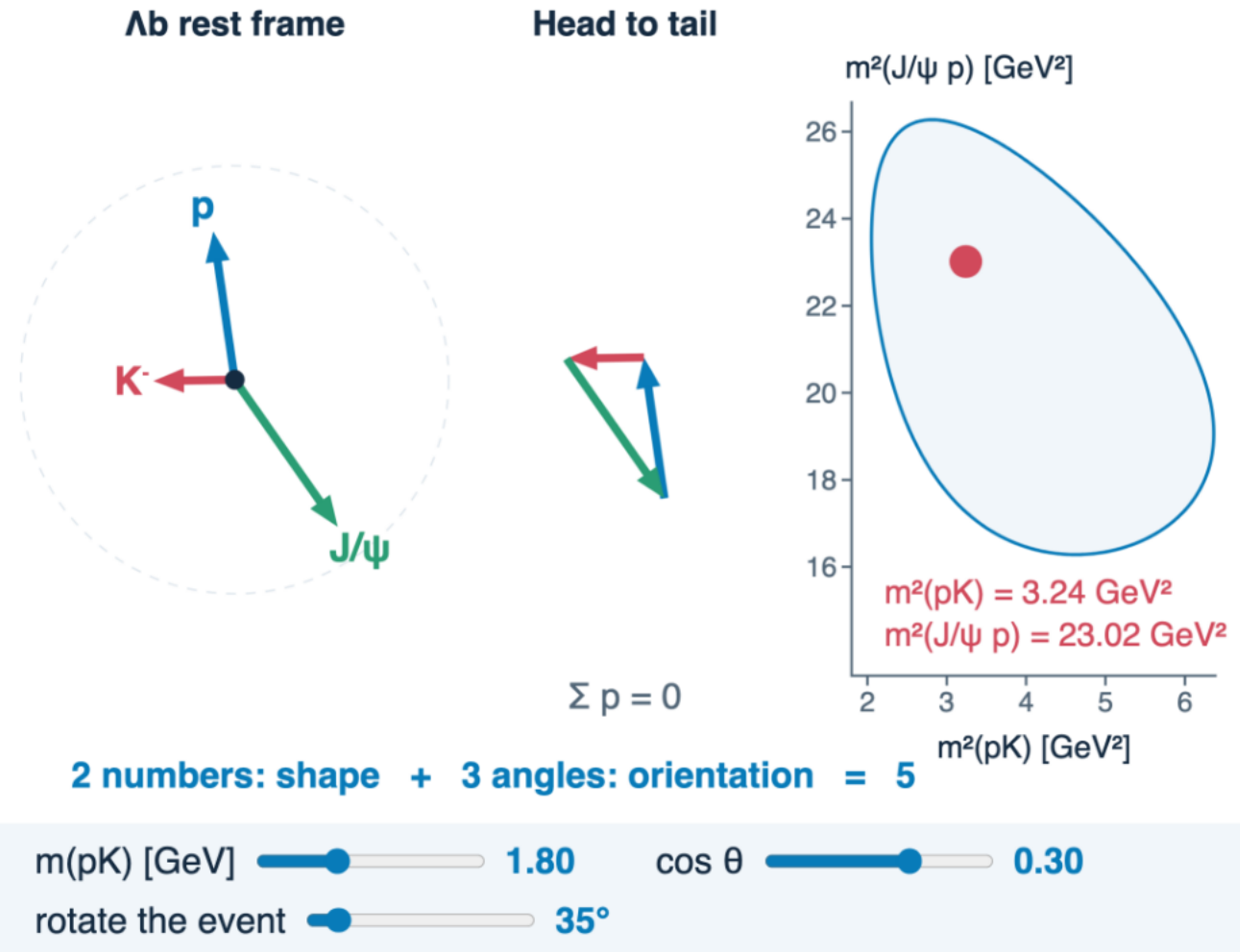
Energy–momentum conservation removes 4.

$$\dim \Phi_n = 3n - 4 = 5$$

Two pair masses fix the shape of the momentum triangle: they span the **Dalitz plot**.

Three Euler angles orient it in space.

Spin projections are labels, not extra coordinates.



Physical masses. Rotating moves every arrow, but no pair mass: the Dalitz point stays put.

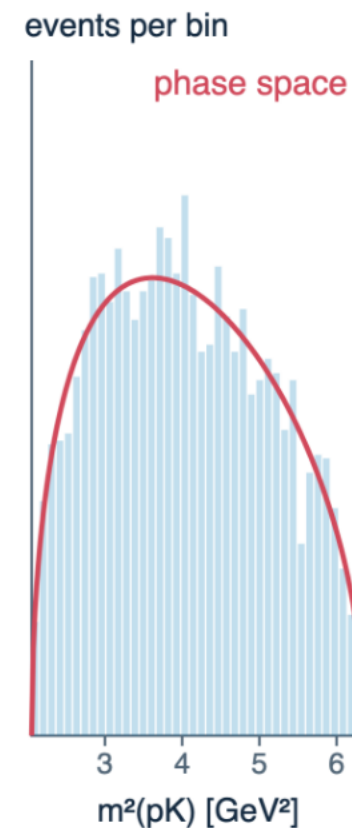
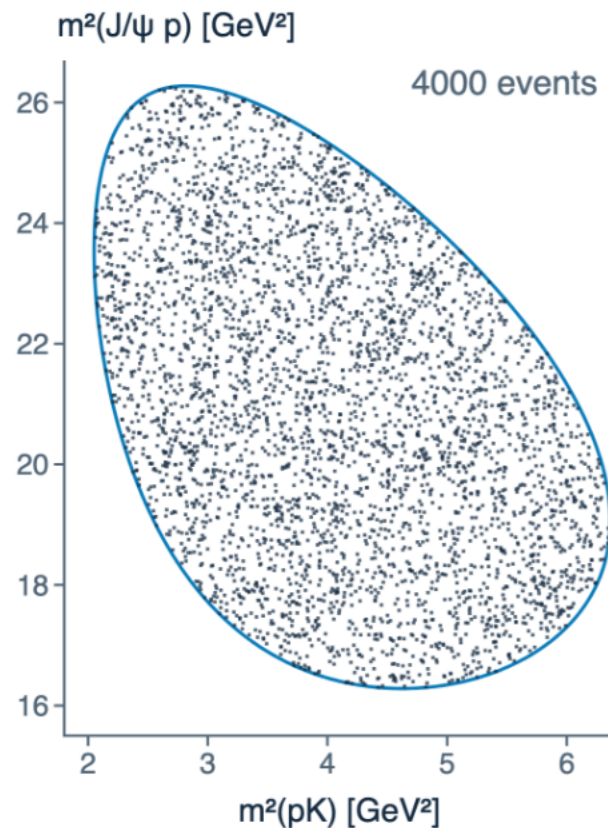
The phase space is flat

Phase-space events fill the plot **uniformly** inside the boundary.

A projection counts the height of the plot at each mass: its curved shape is pure kinematics.

In m rather than m^2 :
 $dN/dm = 2m dN/dm^2$.

Any non-uniformity in the data is dynamics or detector.



events ☒ 4000

☐ project onto $m(pK)$ instead of $m^2(pK)$

Uniform phase-space events: no dynamics, no detector. Red: the exact phase-space projection.

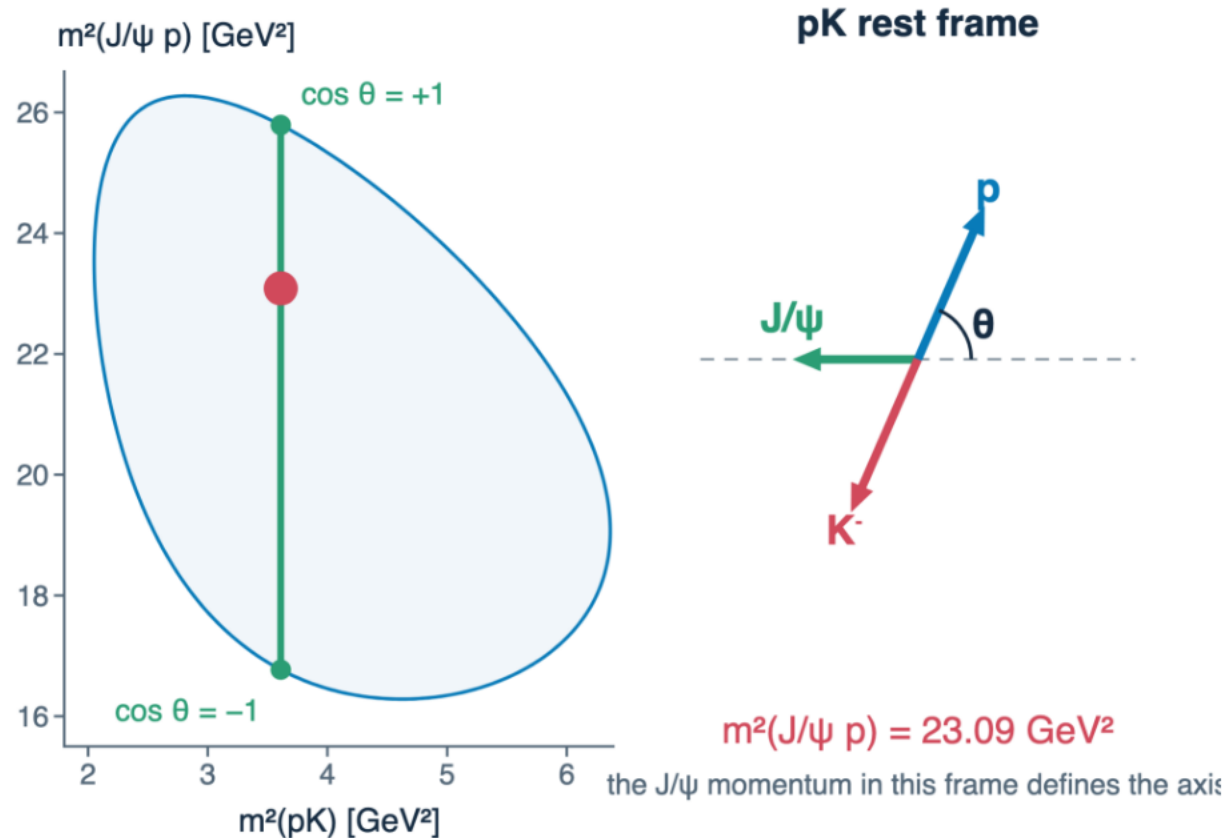
One mass and one angle

Fix $m(pK)$: the event moves along a vertical slice.

The position on the slice is an **angle**: the proton direction in the pK rest frame.

$$m_{J/\psi p}^2 = a + b \cos \theta$$

The Dalitz plot is a mass and an angle. Angular distributions sit along every band.



$$m^2(J/\psi p) = 23.09 \text{ GeV}^2$$

the J/ψ momentum in this frame defines the axis

$m(pK) [\text{GeV}]$ 1.90 $\cos \theta$ 0.40

θ : proton direction in the pK rest frame, measured from the direction opposite J/ψ .

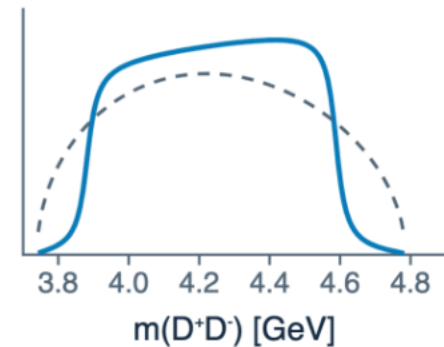
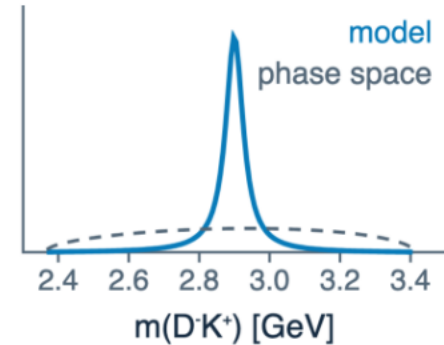
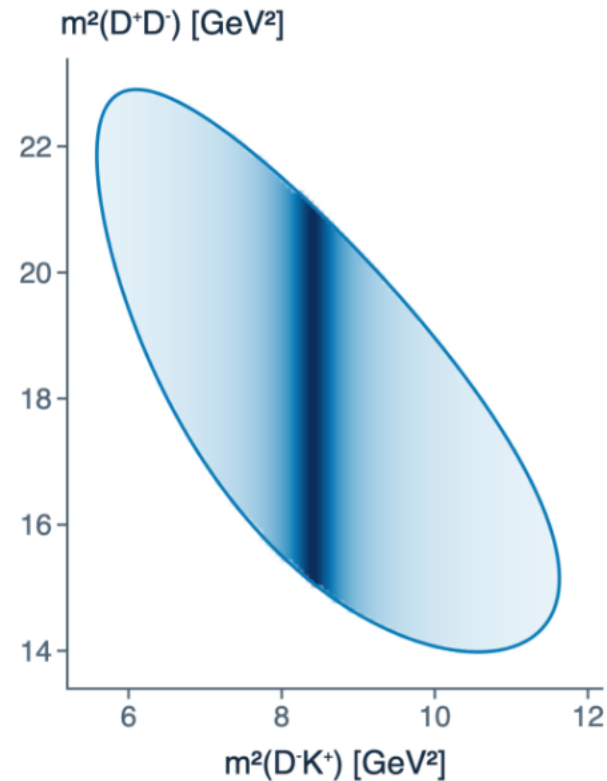
Bands and reflections

Simplest case first: $B^+ \rightarrow D^+ D^- K^+$, **all spin 0**.

A resonance in $D^- K^+$ makes a band at fixed $m(D^- K^+)$.

Projected onto $m(D^+ D^-)$, the same band gives a **broad structure**: a reflection. No $D^+ D^-$ resonance is needed.

Narrow peaks are hard to fake. Broad structures are easy.



D $^-$ K $^+$ mass [GeV] 2.90

width [GeV] 0.06

spin J

$B^+ \rightarrow D^+ D^- K^+$, all spin 0: the angular factor is exactly $P_J(\cos \theta)$. Dashed: phase space.

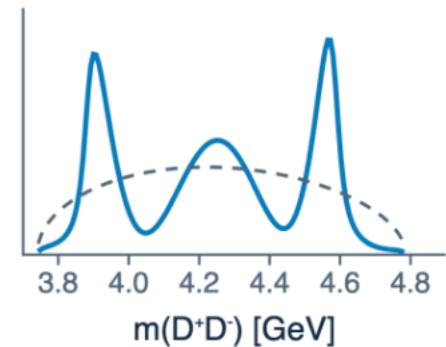
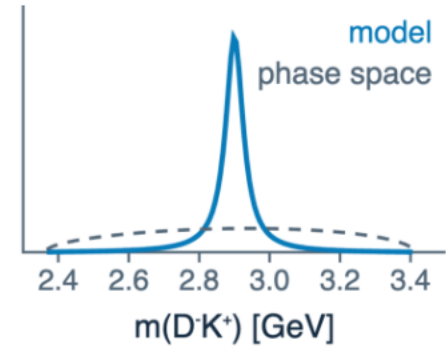
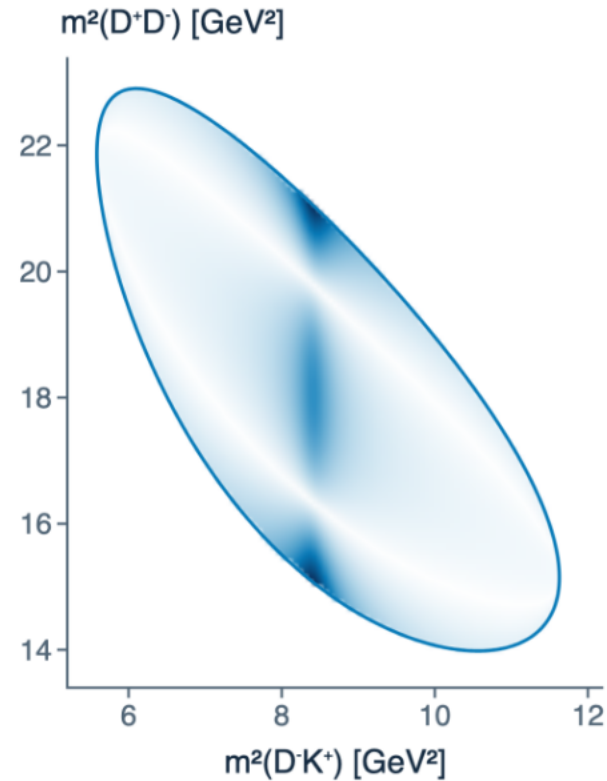
Spin leaves a pattern along a band

Spin- J resonance $\rightarrow$ two spinless particles:

$$\mathcal{A} \propto d_{00}^J(\theta) = P_J(\cos \theta)$$

J zeros along the band.

Spin also shapes the reflection: **bumps** in $m(D^+ D^-)$ without any $D^+ D^-$ state.



D $^+$ K $^+$ mass [GeV]

width [GeV]

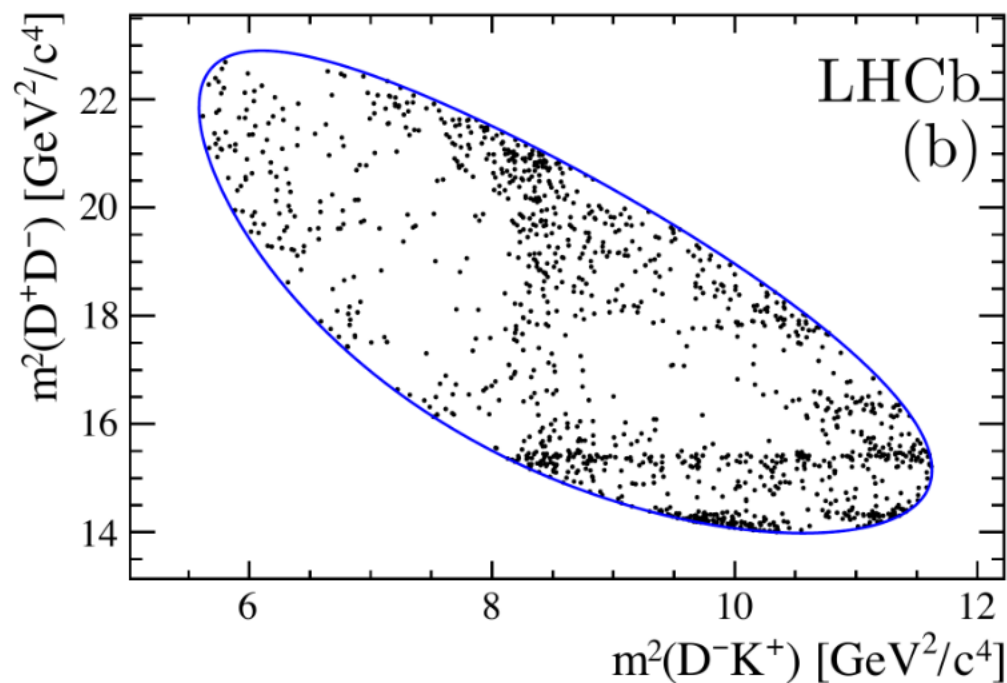
spin J

$B^+ \rightarrow D^+ D^- K^+$, all spin 0: the angular factor is exactly $P_J(\cos \theta)$. Dashed: phase space.

$B^+ \rightarrow D^+D^-K^+$: the data

[LHCb · PRD 102, 112003 \(2020\)](#)

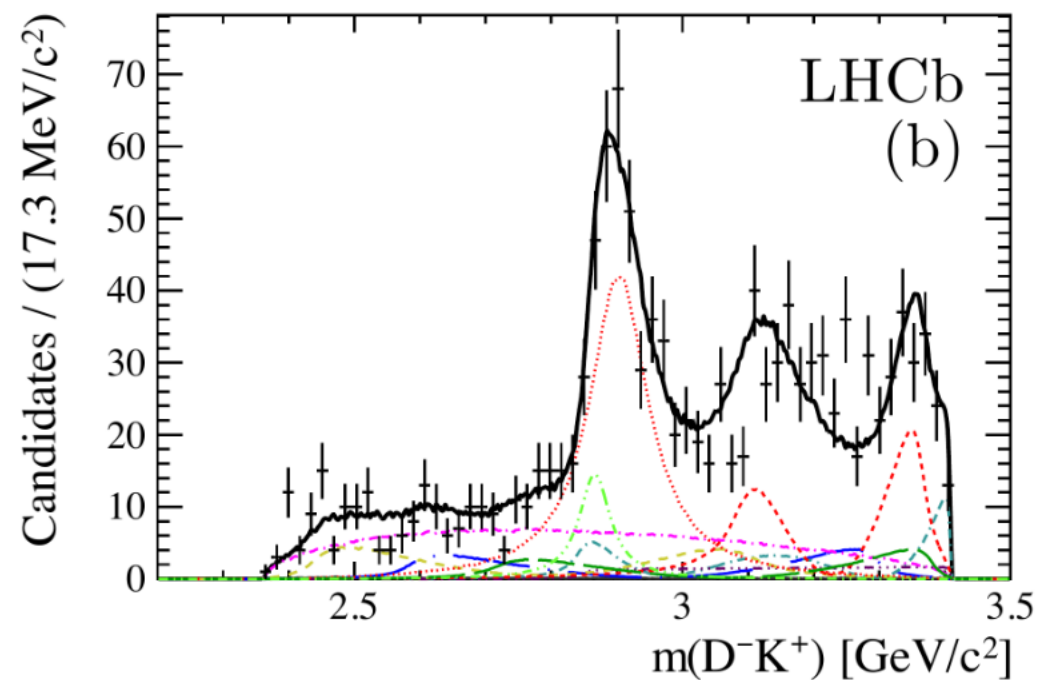
[LHCb · PRL 125, 242001 \(2020\)](#)



Horizontal bands: charmonia in D^+D^-

The pattern along the D^-K^+ band, as in the widget, fixes the spin.

Charmonium bands reflect into $m(D^-K^+)$, but cannot explain the peak at 2.9 GeV.



$m(D^-K^+)$ with the full amplitude fit

The fit needs D^-K^+ states with spin 0 and 1. Quark content $\bar{c}sud$: **manifestly exotic**.

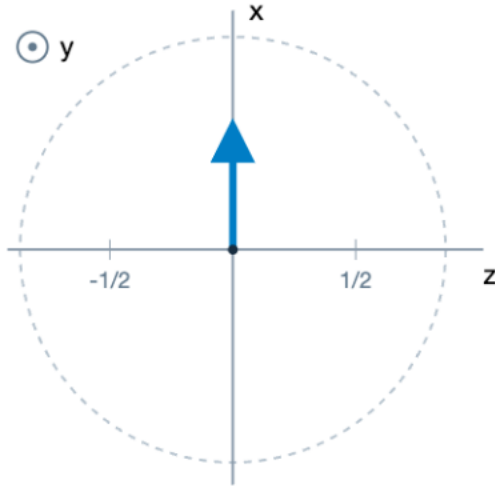
Spin

Angular patterns, decay chains and frames

Spin rotation, revisited

Spin j $\frac{1}{2}$ Rotate ▶ Reset $|j, j\rangle$ θ

90°



$$|\psi(\theta)\rangle = 0.707 |1/2, 1/2\rangle + 0.707 |1/2, -1/2\rangle$$

$$|c_m|^2 =$$

50.0%

50.0%



$$\Sigma |c_m|^2 = 1.000 \cdot 2 \text{ possible values of } m$$

$$d_{m'm}^j(\theta)$$

$d_{m'm}^j(\theta) = \langle j, m' | e^{-i\theta J_y} | j, m \rangle$: the **Wigner d-functions** are the coefficients above. Amplitudes, with signs.

With three Euler angles, $D_{m'm}^j(\alpha, \beta, \gamma) = e^{-im'\alpha} d_{m'm}^j(\beta) e^{-im\gamma}$.

One decay vertex

For $A(J, m) \rightarrow 1 + 2$ with helicities λ_1, λ_2 :

$$\mathcal{A}_{m; \lambda_1 \lambda_2} = \underbrace{D_{m, \lambda_1 - \lambda_2}^{J*}}_{\text{rotations: fixed}}(\phi, \theta, 0) \underbrace{H_{\lambda_1 \lambda_2}}_{\text{fitted}}$$

Helicity: spin along the particle's momentum.

The angles are those of particle 1 in the rest frame of A.

How many couplings H?

One per allowed (L, S) combination.

Strong $\Lambda(1520) [\frac{3}{2}^-] \rightarrow p K^-$: parity allows only $L = 2 \rightarrow \mathbf{1}$ coupling.

Weak $\Lambda_b \rightarrow \Lambda^* J/\psi$: no parity constraint $\rightarrow \mathbf{4}$ couplings ($J = \frac{1}{2}$) or $\mathbf{6}$ ($J \geq \frac{3}{2}$).

Each is a complex number to fit.

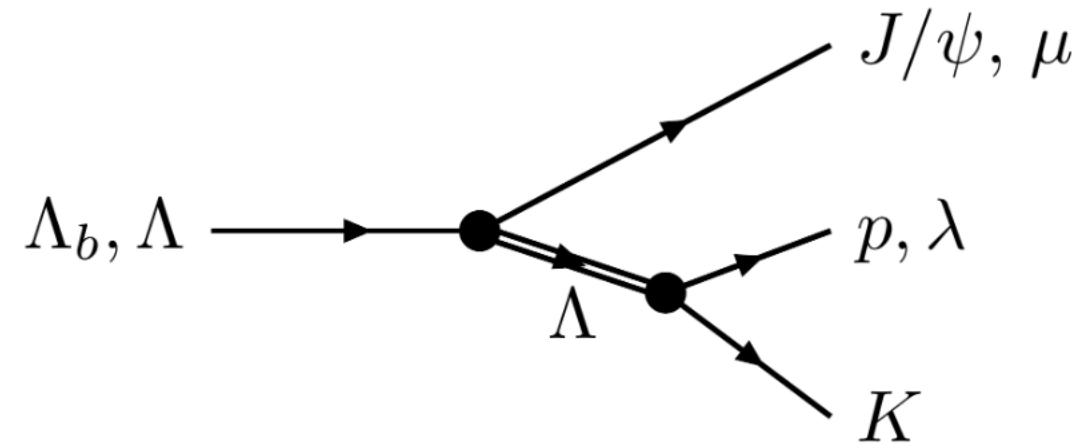
A decay chain

$\Lambda_b \rightarrow \Lambda^* J/\psi$, then $\Lambda^* \rightarrow pK^-$:

$$\mathcal{A}^{\Lambda^*} = \sum_v \mathcal{A}_v^{\Lambda_b \rightarrow \Lambda^* \psi} \text{BW}(m_{pK}^2) \mathcal{A}_v^{\Lambda^* \rightarrow pK}$$

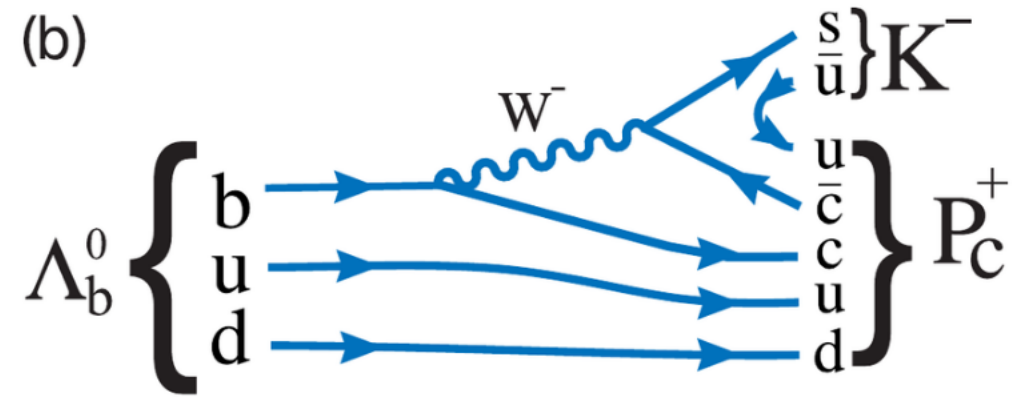
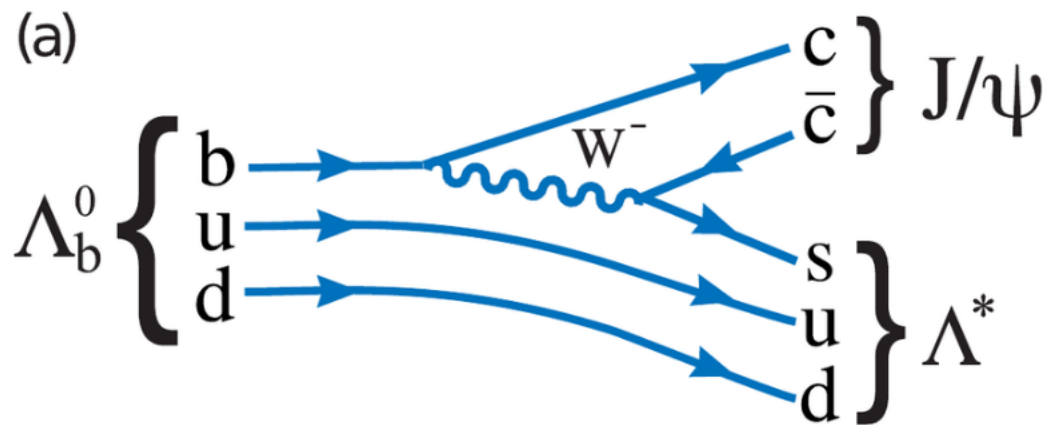
Two vertices, each in its own rest frame, joined by a propagator.

The Λ^* helicity v is not observed and is internal:
sum the **amplitudes**.



Labels Λ, μ, λ beside the external lines are spin projections; the internal Λ is the Λ^* .

Two chains, one final state



Λ^* chain: $b \rightarrow c\bar{c}s$; the s quark forms an excited Λ^* with ud .

P_c chain: the $c\bar{c}$ pair stays with uud and recoils against K^- .

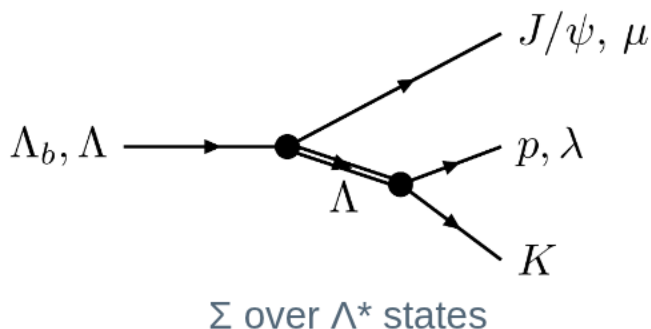
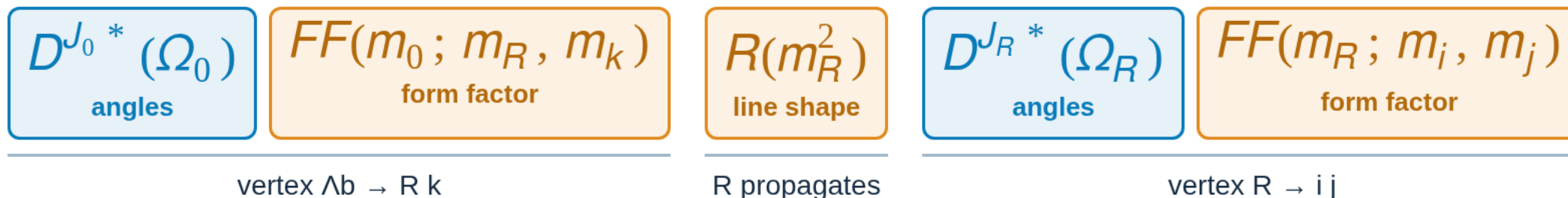
Same particles, same momenta: the amplitudes **add**, then we square.

The full amplitude

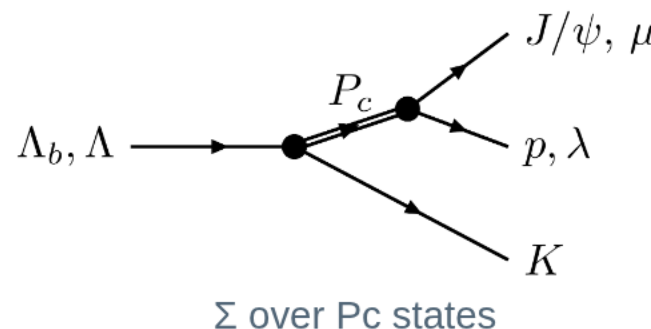
JPAC · PRD 101, 034033 (2020)

LHCb · PRL 115, 072001 (2015)

One decay chain: two vertices and a propagator.



$$\mathcal{A}_\lambda = \sum_{\text{chains } k} c_k A_\lambda^{(k)}$$



$$I = \sum_\lambda |\mathcal{A}_\lambda|^2$$

■ **Rotations:** fixed by spin (this section) ■ **Mass dependence:** model choices (next section) ■ c_k : **complex couplings**, fitted

Coherent over chains, incoherent over unobserved spins. P_c chains also need the spin alignment $d(\zeta)$ (backup).

Frames, live

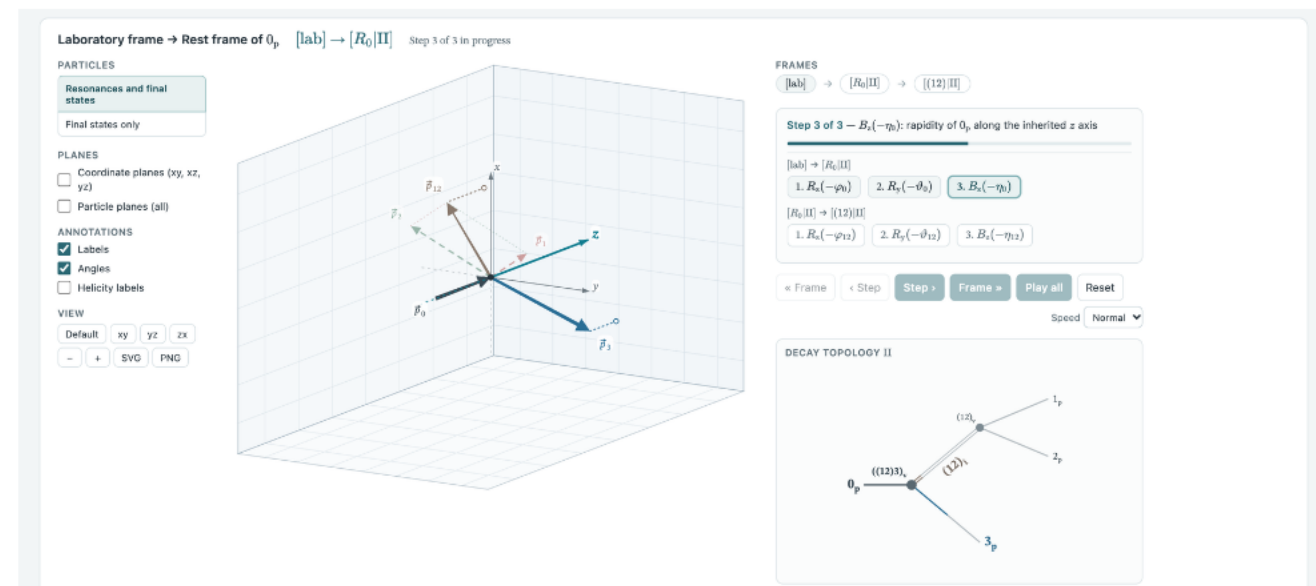
Start in the laboratory: the Λ_b flies.

Rotate, then boost, into the Λ_b rest frame.

Repeat for the pK^- pair: its rest frame, with the z axis along its flight direction.

Every angle in the amplitude is measured in such a frame.

[Open the Dynamic view](#)



Mass dependence

Line shapes and vertex form factors

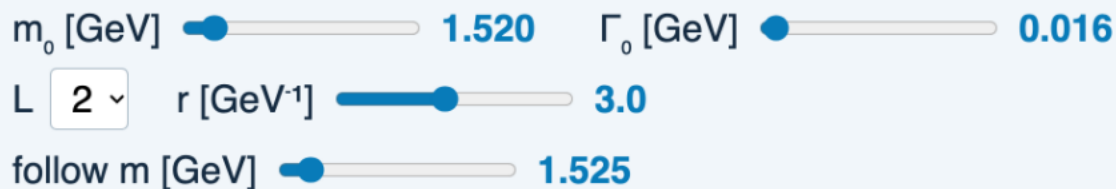
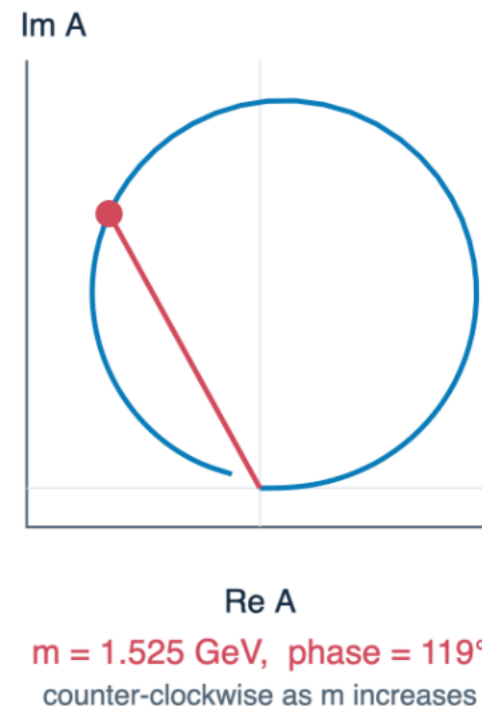
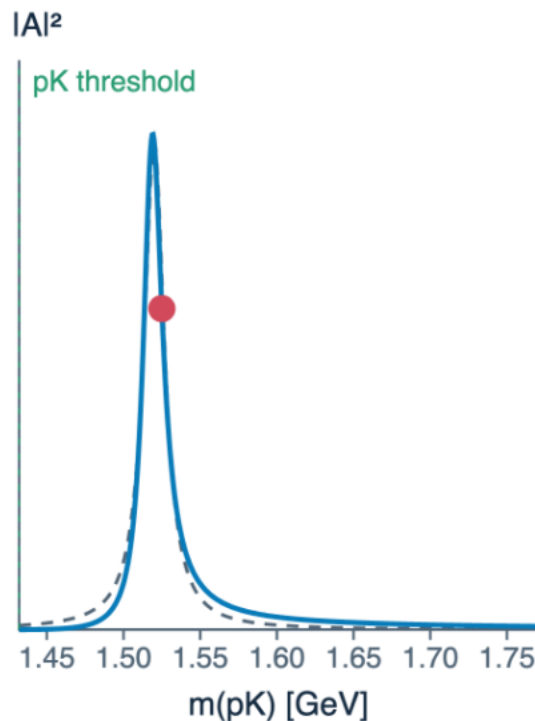
The resonance line shape

$R(m_R^2)$: how the resonance propagates.
The typical construction is a Breit–Wigner with a running width:

$$R(\sigma) = \frac{1}{m_R^2 - \sigma - i m_R \Gamma(\sigma)}$$

$$\Gamma(\sigma) = \Gamma_R \frac{q}{q_R} \frac{m_R}{\sqrt{\sigma}} FF_L^2(\sigma)$$

A peak in $|R|^2$, and a phase that rises through 90° (Lecture 1). The width carries the decay form factor (next slide).



$\Lambda(1520) \rightarrow p\bar{K}$ in D wave. Solid: running width and barrier factors; dashed: constant width.

Form factors

PDG · Resonances (2025)

von Hippel & Quigg · PRD 5, 624 (1972)

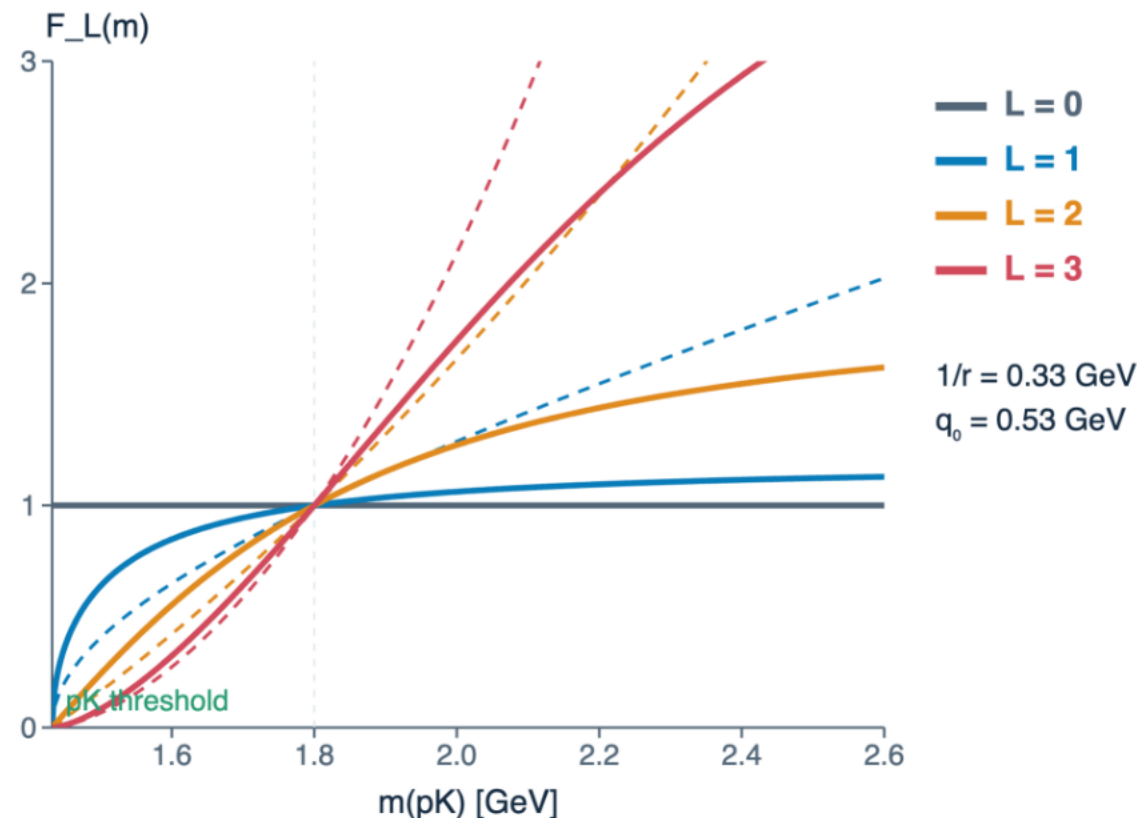
$FF(m_0; m_1, m_2)$: the vertex where $m_0 \rightarrow m_1 + m_2$, with breakup momentum q .

$$FF_L = \left(\frac{q}{q_0}\right)^L \frac{B_L(qr)}{B_L(q_0 r)}$$

q^L : the angular-momentum barrier.
Partial wave L is suppressed near threshold.

Blatt-Weisskopf B_L : stops the growth for $q \gtrsim 1/r$, with $r \approx 1.5\text{--}5 \text{ GeV}^{-1}$ (0.3–1 fm).

$B_1 = 1/\sqrt{1+z^2}$, $B_2 = 1/\sqrt{9+3z^2+z^4}$, $z = qr$. Two vertices, two form factors: production uses p , decay uses q .



$r [\text{GeV}^{-1}]$ 3.0

normalise at $m_0 [\text{GeV}]$ 1.80

☒ show bare q^L

$R \rightarrow pK$: $F_L(m) = (q/q_0)^L B_L(qr)/B_L(q_0 r)$, equal to 1 at m_0 . Dashed: $(q/q_0)^L$ alone.

The full model: $\Lambda_b \rightarrow J/\psi \, p \, K^-$

Spin, interference and the pentaquarks

With a proton, the zeros fill in

JPAC · PRD 101, 034033 (2020)

[ThreeBodyDecays.jl](#)

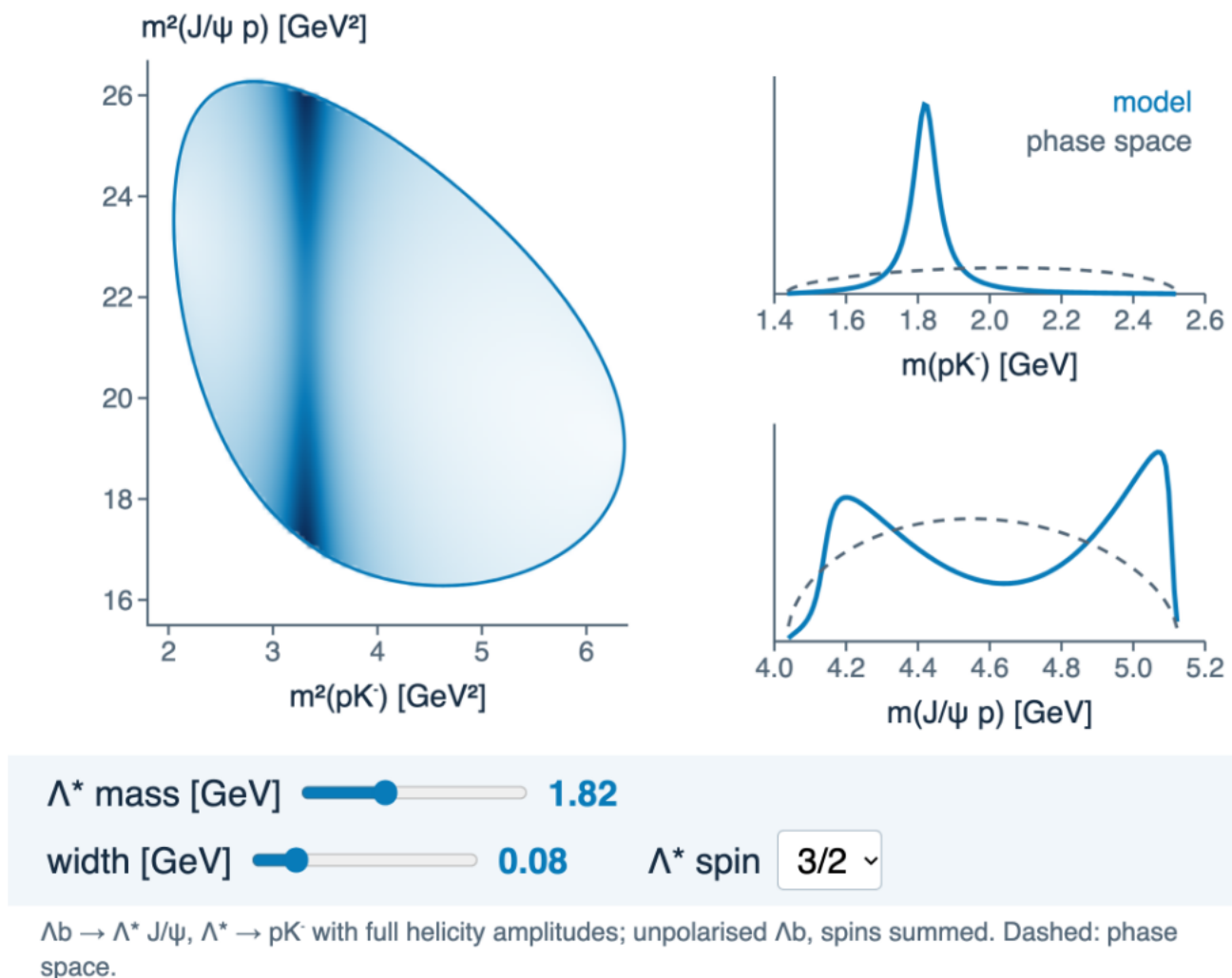
Back to $\Lambda^* \rightarrow pK^-$: the proton has two helicities, $\pm\frac{1}{2}$.

Its spin is not measured: add $|A|^2$ for both.

Each term has zeros; the sum does not.
For $J = \frac{3}{2}$:

$$|d_{\frac{1}{2}, \frac{1}{2}}^{3/2}|^2 + |d_{\frac{1}{2}, -\frac{1}{2}}^{3/2}|^2 \propto 1 + 3 \cos^2 \theta$$

Spin is still visible, but less sharply.
LHCb also used the $J/\psi \rightarrow \mu^+\mu^-$ angles.



Unpolarised and polarised parents

[JPAC · PRD 101, 034033 \(2020\)](#)

[LHCb · PLB 724, 27 \(2013\)](#)

[LHCb · \$\Lambda_c^+\$ polarimetry, JHEP 07, 228 \(2023\)](#)

[ComPWA · polarimetry](#)

The parent's spin states form an ensemble, described by a density matrix:

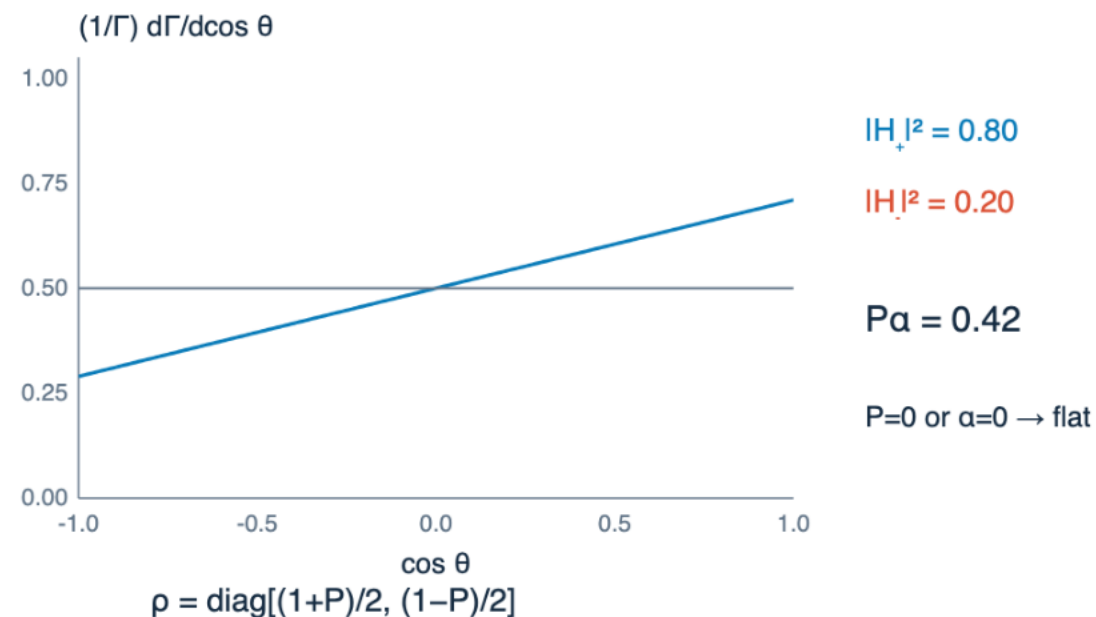
$$I(x) = \sum_{\text{final spins}} \sum_{m, m'} \rho_{mm'} \mathcal{A}_m \mathcal{A}_{m'}^*$$

Unpolarised ($\rho \propto 1$): no preferred direction. Integrate the orientation; the Dalitz variables remain.

Polarised: the **production angles** — the orientation of the decay relative to the polarisation axis — become anisotropic: $1 + \alpha P \cos \theta$.

Λ_b at LHCb: P compatible with zero. $\Lambda_c^+ \rightarrow pK^-\pi^+$ as a polarimeter: [paper](#), [website](#).

A spin- $\frac{1}{2}$ decay can analyse parent polarisation



Parent P 0.7 Decay α 0.6

Spin- $\frac{1}{2} \rightarrow \text{spin-}\frac{1}{2} + \text{spin-}0$, with daughter spin unobserved. $\alpha = (|H_+|^2 - |H_-|^2) / (|H_+|^2 + |H_-|^2)$. Strong parity conservation forces $\alpha=0$; weak decays may have $\alpha \neq 0$. Values are illustrative, not measured Λ parameters.

Interference reshapes the plot

[JPAC · PRD 101, 034033 \(2020\)](#)

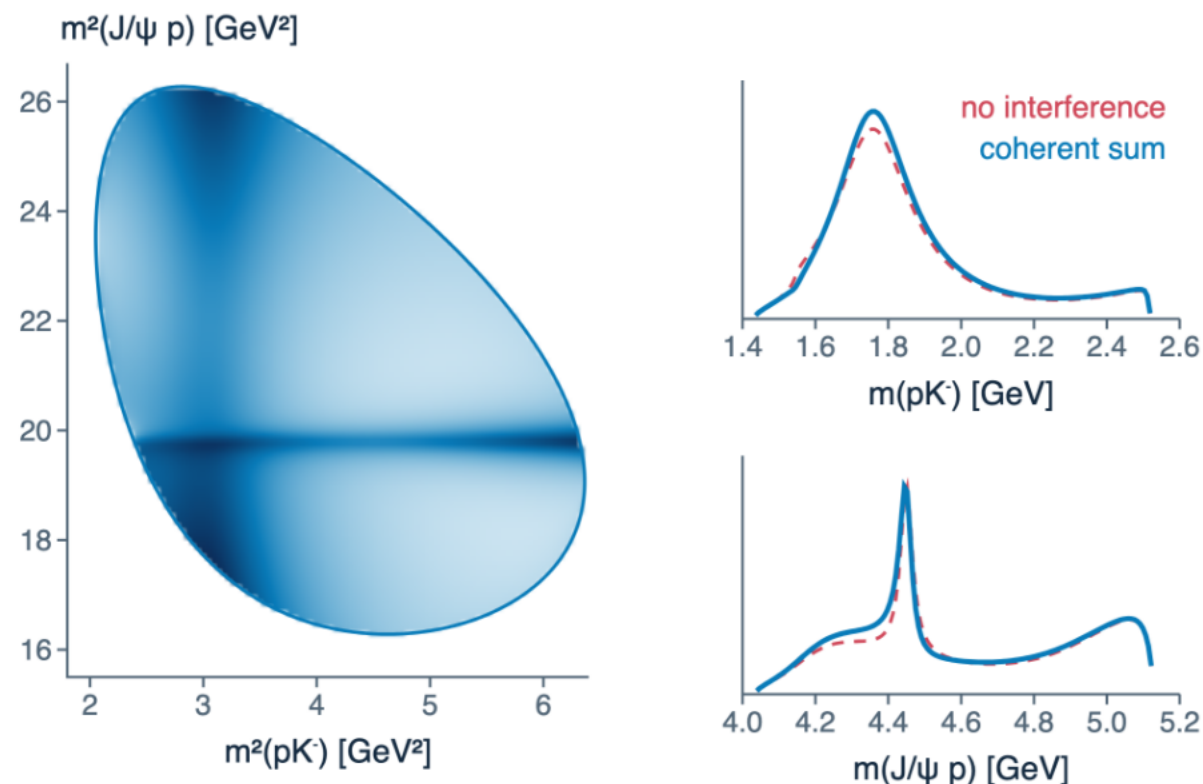
[ThreeBodyDecays.jl](#)

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2 \operatorname{Re} A_1 A_2^*$$

The cross term follows the **relative phase**: the Dalitz pattern and the Pc line shape change.

Summing over spins dilutes it, but does not remove it.

The projection alone cannot tell. The fit measures the phase.



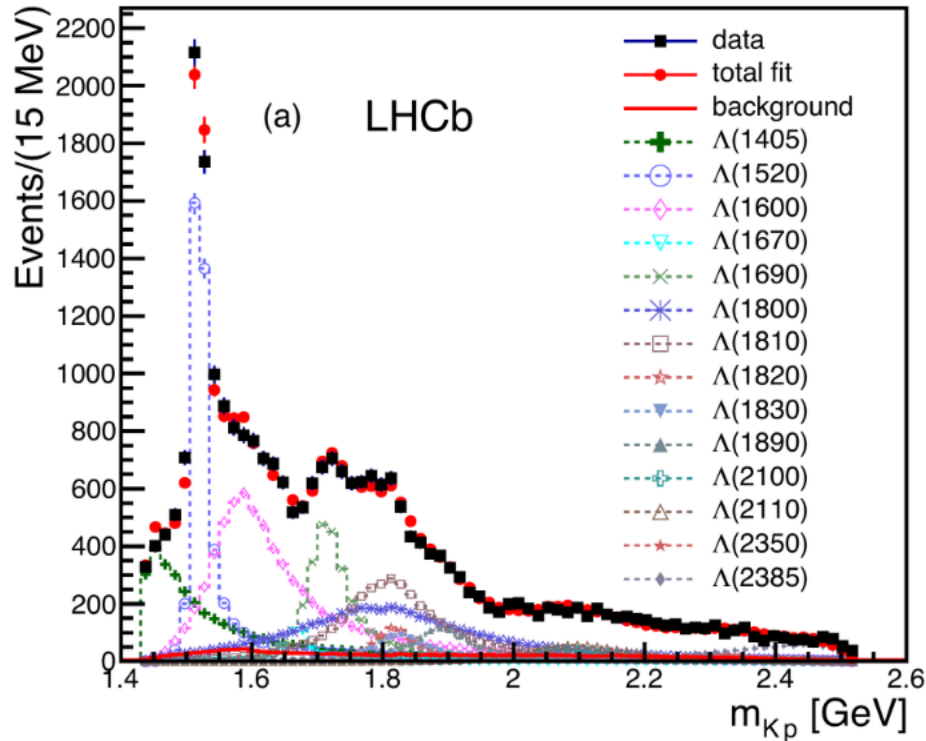
relative phase ϕ 1.60

Pc strength 1.00

☒ add amplitudes (coherent)

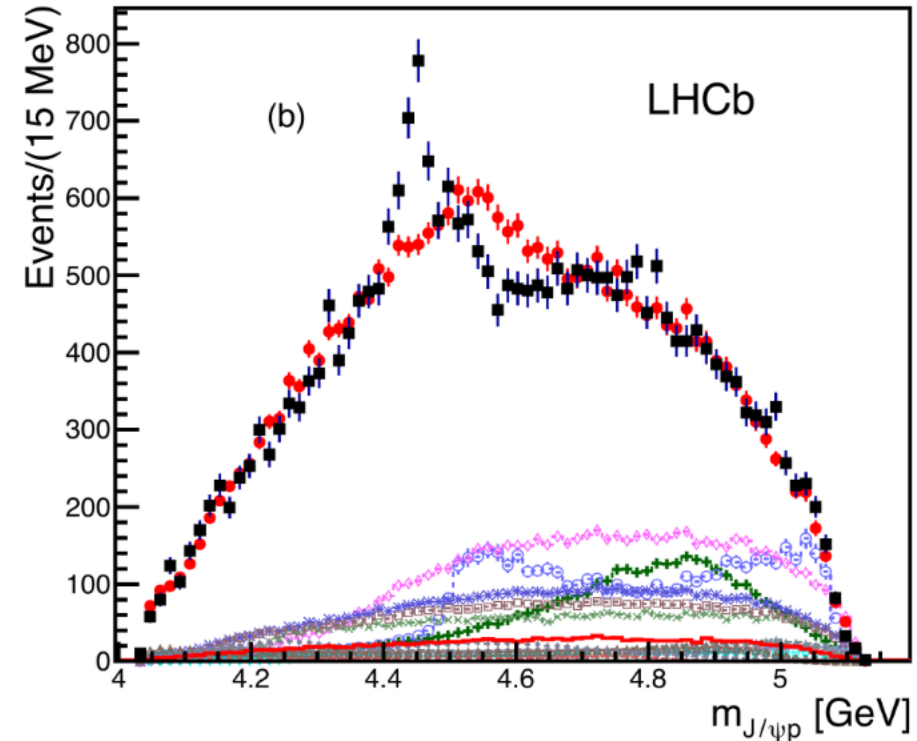
$\Lambda^*(3/2^-) + Pc(3/2^-)$ in $\Lambda b \rightarrow J/\psi p K^-$: full helicity amplitudes (ThreeBodyDecays.jl formulas), illustrative couplings. Unticked: no interference.

Λ^* states alone fail



$m(Kp)$: the Λ^* states describe their own channel

Fit with all 14 Λ^* states and every allowed coupling.



$m(J/\psi p)$: the peak is missed

The reflections are included, with all interference.

The narrow structure near 4.45 GeV is still not described.

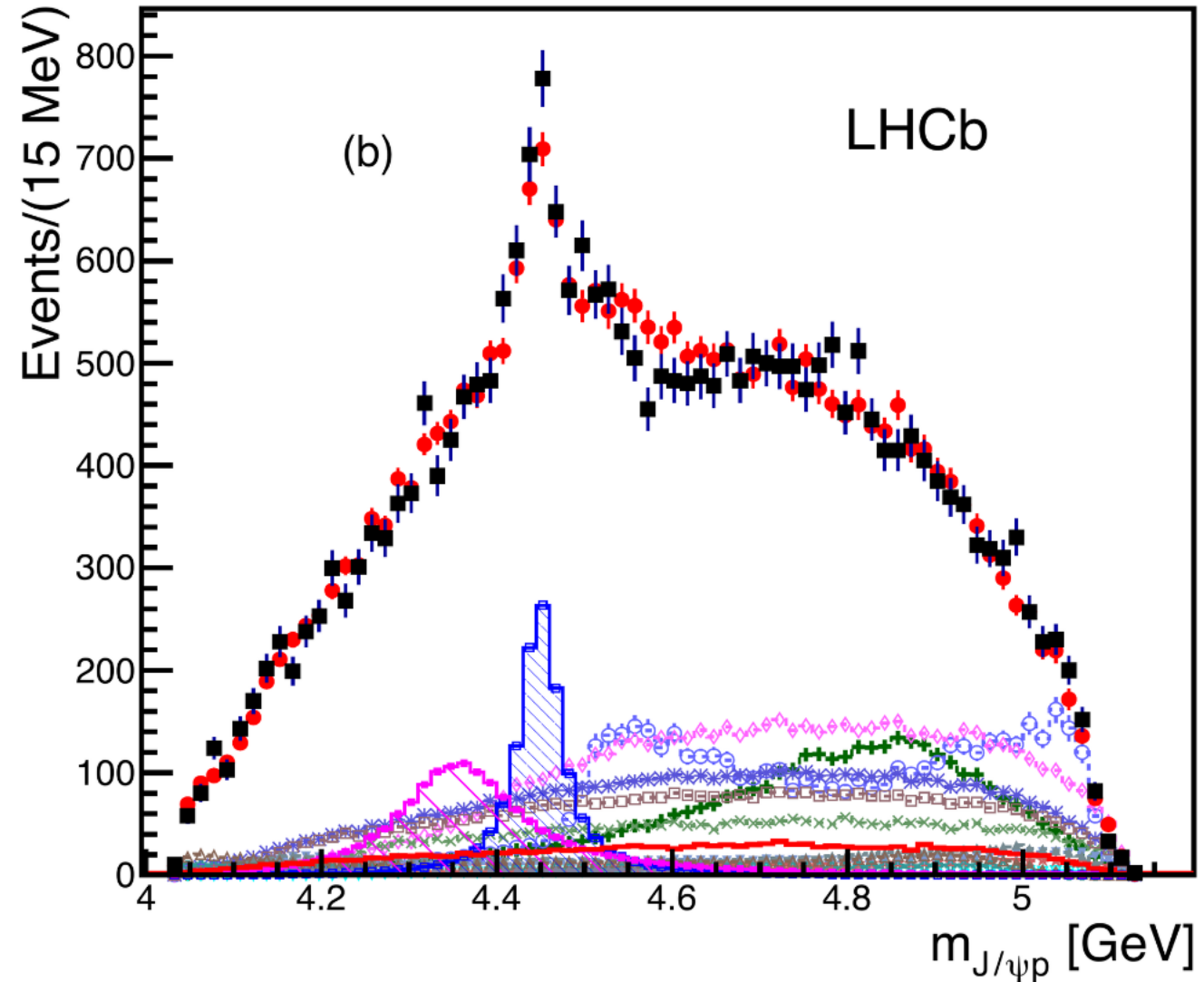
Adding Pc states

Add two $J/\psi p$ states to the Λ^* model:
now the data are described.

A narrow $P_c(4450)^+$ ($\Gamma \approx 40$ MeV) and a
broad $P_c(4380)^+$ ($\Gamma \approx 200$ MeV). Stat.
significance: 12σ and 9σ .

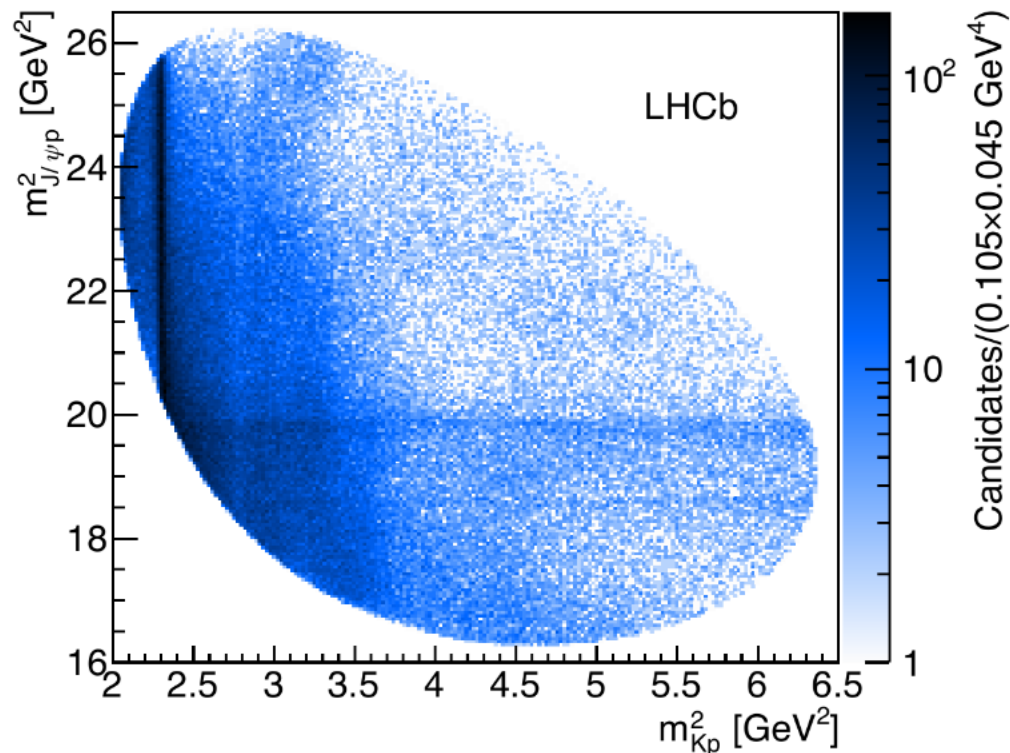
Opposite parities; best spins $\frac{3}{2}$ and $\frac{5}{2}$,
from the decay angles.

There was more to discover...

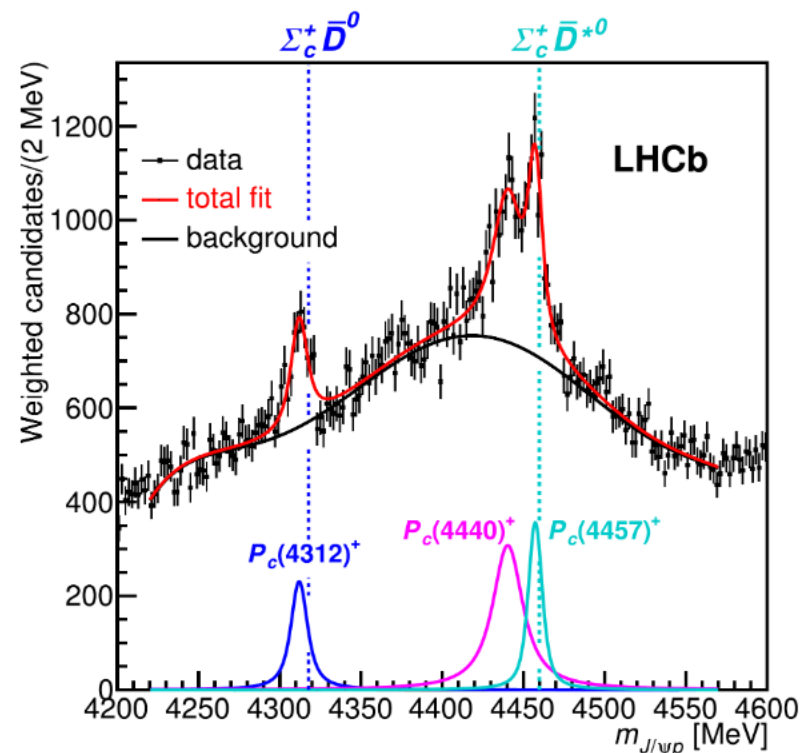


2019: more data, a new picture

LHCb · PRL 122, 222001 (2019)



Nine times more Λb decays



Three narrow peaks in $m(J/\psi p)$

The 4450 structure splits into $P_c(4440)^+$ and $P_c(4457)^+$. A new $P_c(4312)^+$ appears.

A fit of the mass projection: spins are not determined; the broad $P_c(4380)^+$ neither confirmed nor excluded.

Pentaquarks: measured and interpreted

LHCb · PRL 122, 222001 (2019)

Hanhart & MM · arXiv:2606.10634

Measured

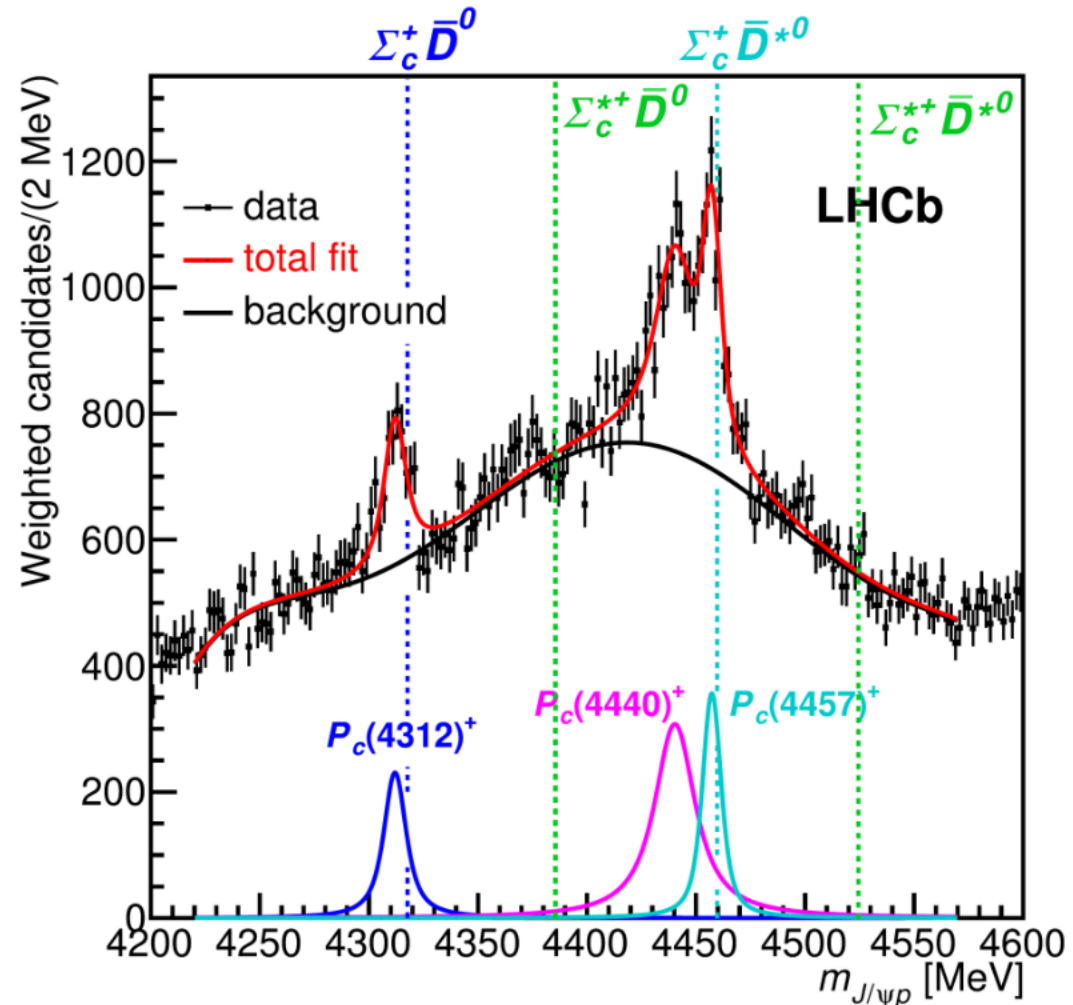
Three narrow peaks: $P_c(4312)^+$, $P_c(4440)^+$, $P_c(4457)^+$, widths 6–20 MeV.

Each sits 2–20 MeV below a $\Sigma_c^+ \bar{D}^0$ or $\Sigma_c^+ \bar{D}^{*0}$ threshold. Spins not yet determined.

Interpreted

$\Sigma_c \bar{D}^{(*)}$ molecules give this pattern naturally (Lecture 1). Compact pentaquarks and threshold effects are also discussed.

Deciding needs spins, partner states and poles — from amplitude analyses.



2027: the full amplitude analysis

[LHCb · PRL 122, 222001 \(2019\)](#)

The full amplitude analysis of the Run 1 + 2 sample is about to be finished.

It is very complicated.

Why it takes so long

High statistics: small effects become significant; the model must describe them all.

Coupled-channel dynamics: narrow states at the $\Sigma c \bar{D}^{(*)}$ thresholds need more than Breit–Wigners.

Fixes in the formalism: interference between chains needs consistent spin alignment.

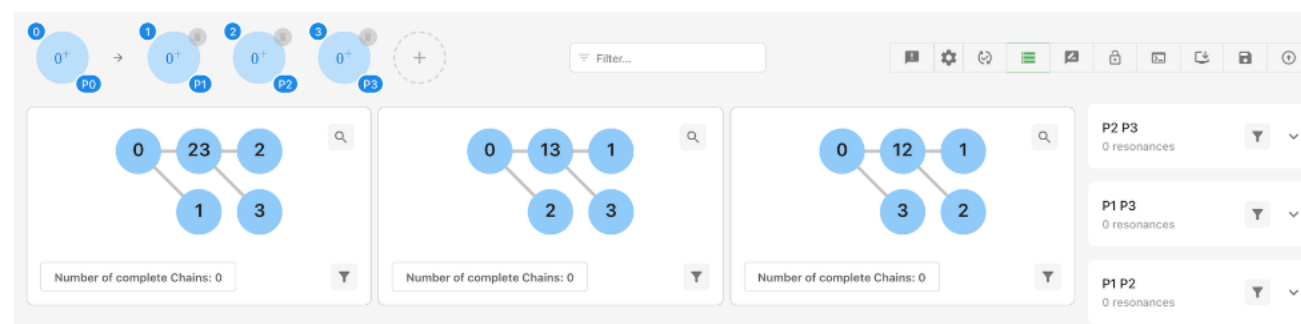
Tools for amplitude models

DecaySelector: declare the particles and their J^P ; it lists the three decay topologies, collects the resonances in each pair, and counts the complete decay chains.

Decay-Web-Page: frames, rotations and the amplitude of each chain.

ThreeBodyDecays.jl: the amplitudes used in today's widgets.

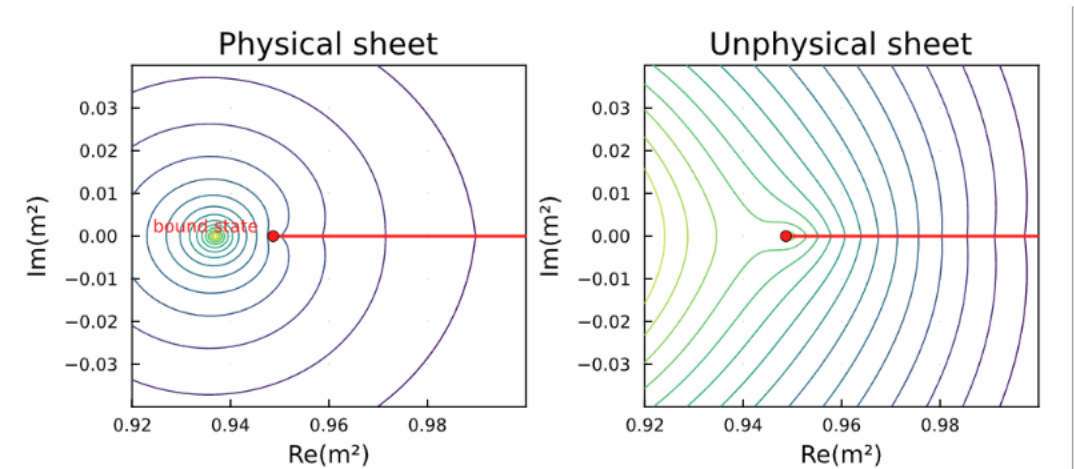
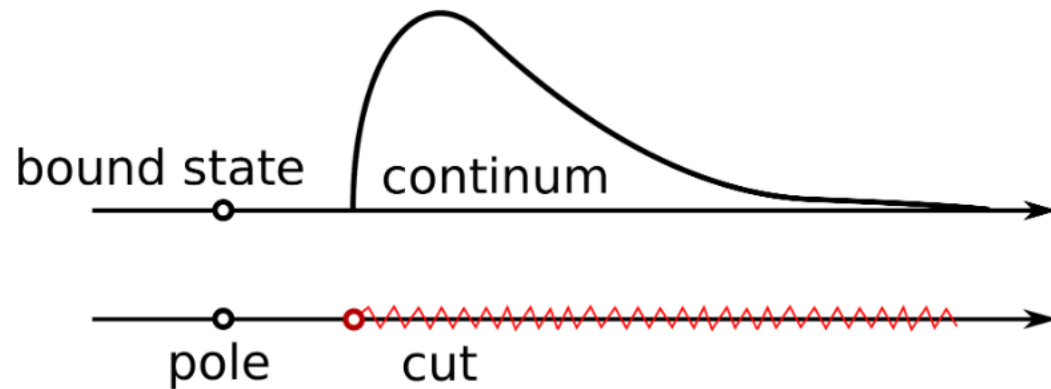
[Open DecaySelector](#)



Near threshold: T_{cc}^+

When a Breit–Wigner is not enough

A bound state below threshold



Bound state: a pole below the continuum

Weaker attraction: the pole moves (Lecture 1)

Below threshold, a bound state is a **pole** of the amplitude. Above it, free $D^* D$ pairs form the continuum: a **cut**.

The weaker the binding, the closer the pole to the threshold; then it becomes virtual.

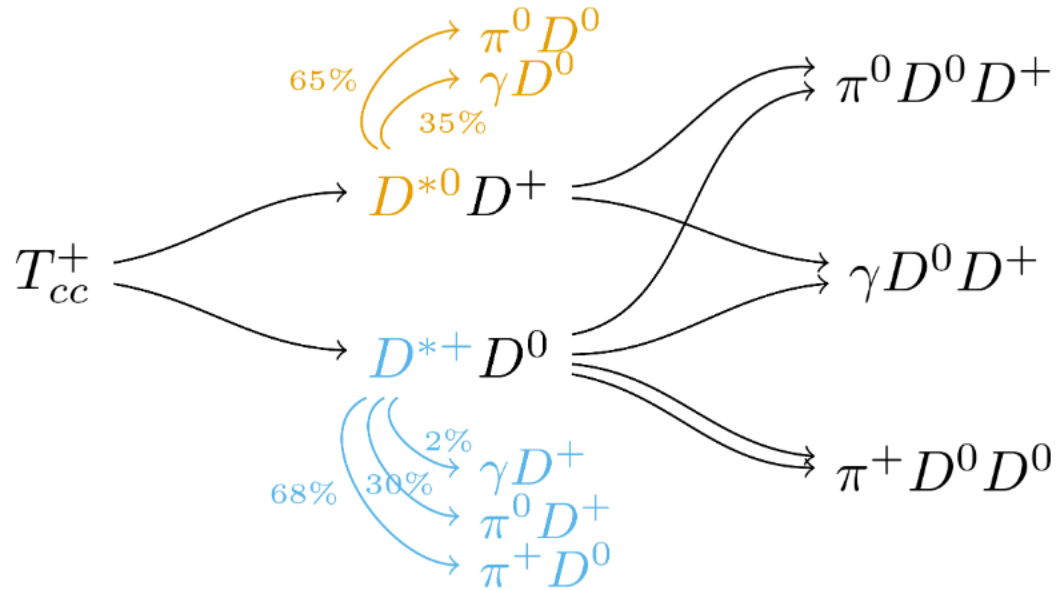
A shallow bound state is large: $\psi \propto e^{-\kappa r}/r$, $\kappa = \sqrt{2\mu B}$. For T_{cc}^+ , $B \approx 0.36$ MeV gives $1/\kappa \approx 7.5$ fm.

The T_{cc}^+ decay amplitude

LHCb · Nature Phys. 18, 751 (2022)

LHCb · Nature Commun. 13, 3351 (2022)

MM et al. (JPAC) · JHEP 08, 080 (2019)

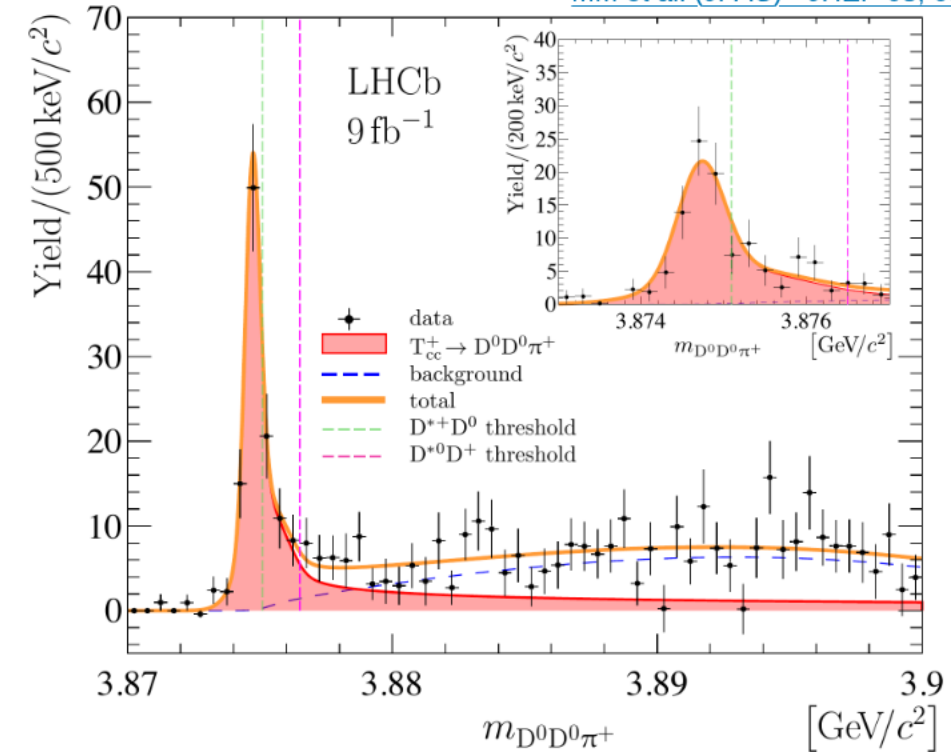


Two $D^* D$ channels, three final states

$J^P = 1^+$: **S-wave** decay to $D^* D$. An isoscalar: equal couplings to $D^{*+} D^0$ and $D^{*0} D^+$.

The model is **unitary**: the D^* width and all three-body channels enter the line shape. Two parameters: peak position and coupling.

Pole: $\delta m = -360 \pm 40$ keV, $\Gamma \approx 48$ keV, below the $D^{*+} D^0$ threshold. A constant-width Breit–Wigner misses the threshold.

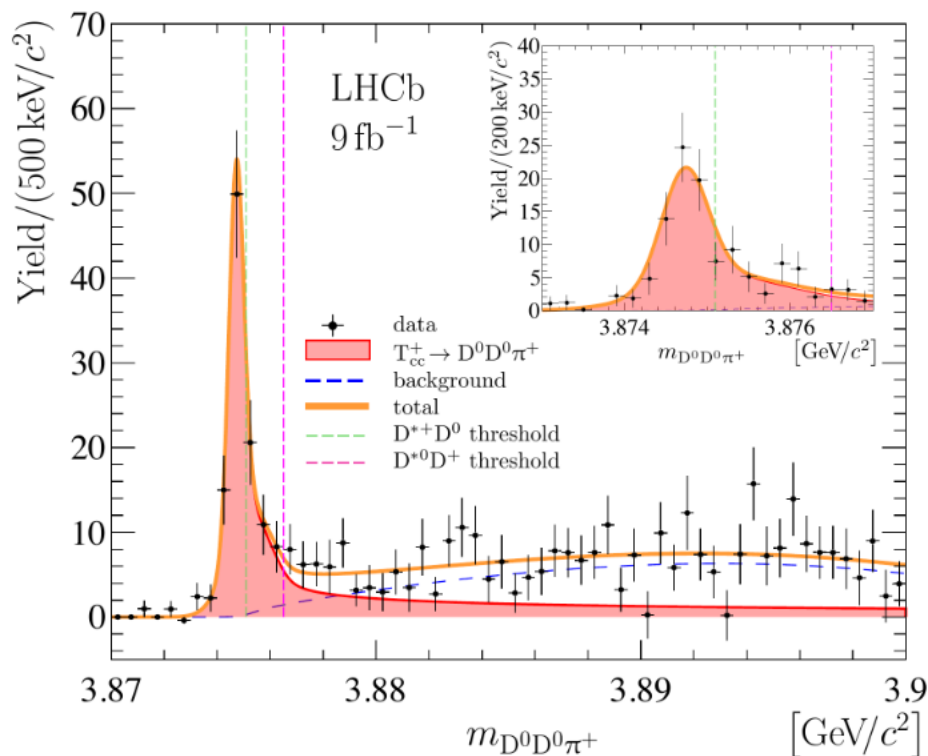


The unitarised model fit to $D^0 D^0 \pi^+$

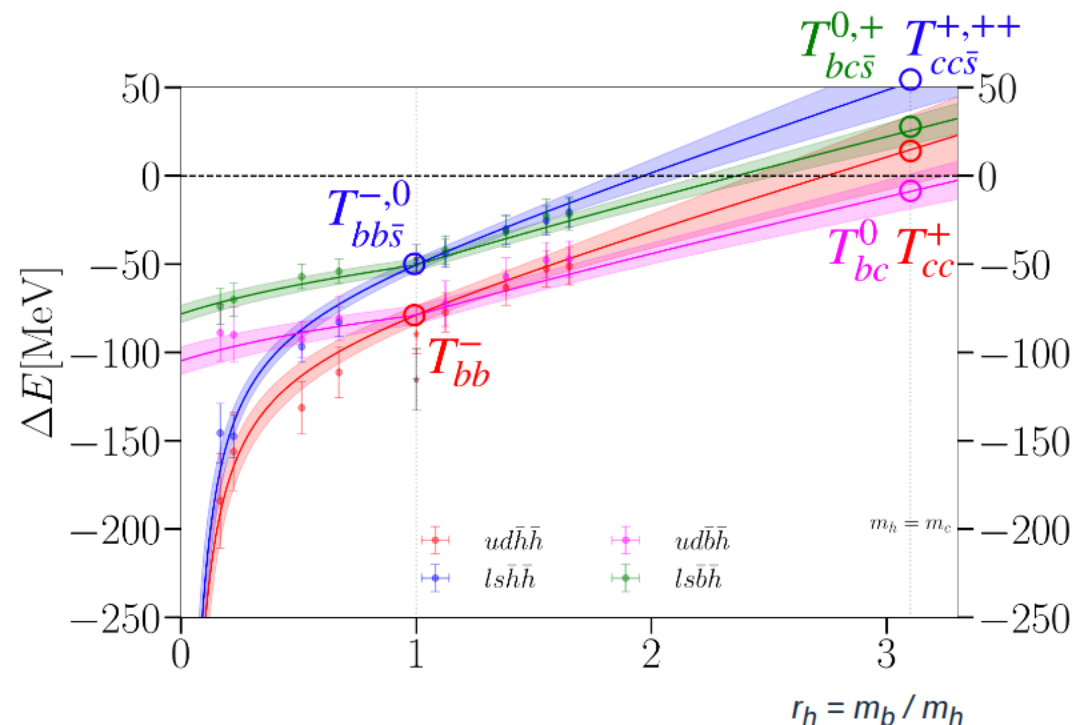
Tcc and Tbb: experiment and lattice

LHCb · Nature Phys. 18, 751 (2022)

Colquhoun et al. · PRD 110, 094503



Experiment: T_{cc}^+ , barely bound



Lattice QCD: binding versus heavy-quark mass

Lattice QCD at $m_\pi \approx 300$ MeV: binding energy versus heavy-quark mass; bottom at $r_h = 1$, charm at $r_h \approx 3$.

Heavier quarks bind deeper: T_{bb} predicted far below threshold, stable against strong decay.

At the charm mass the lattice point is near threshold; the measured T_{cc} is 0.36 MeV below. T_{bc} has a good chance to be discovered at the LHC.

What we have discussed

Kinematics

Five numbers per three-body event: two for the shape (the Dalitz plot), three for the orientation.

Phase space is flat; bands, reflections and spin zeros are dynamics.

Spin

Each vertex: rotation $\times$ coupling. Chains with the same final state add as amplitudes.

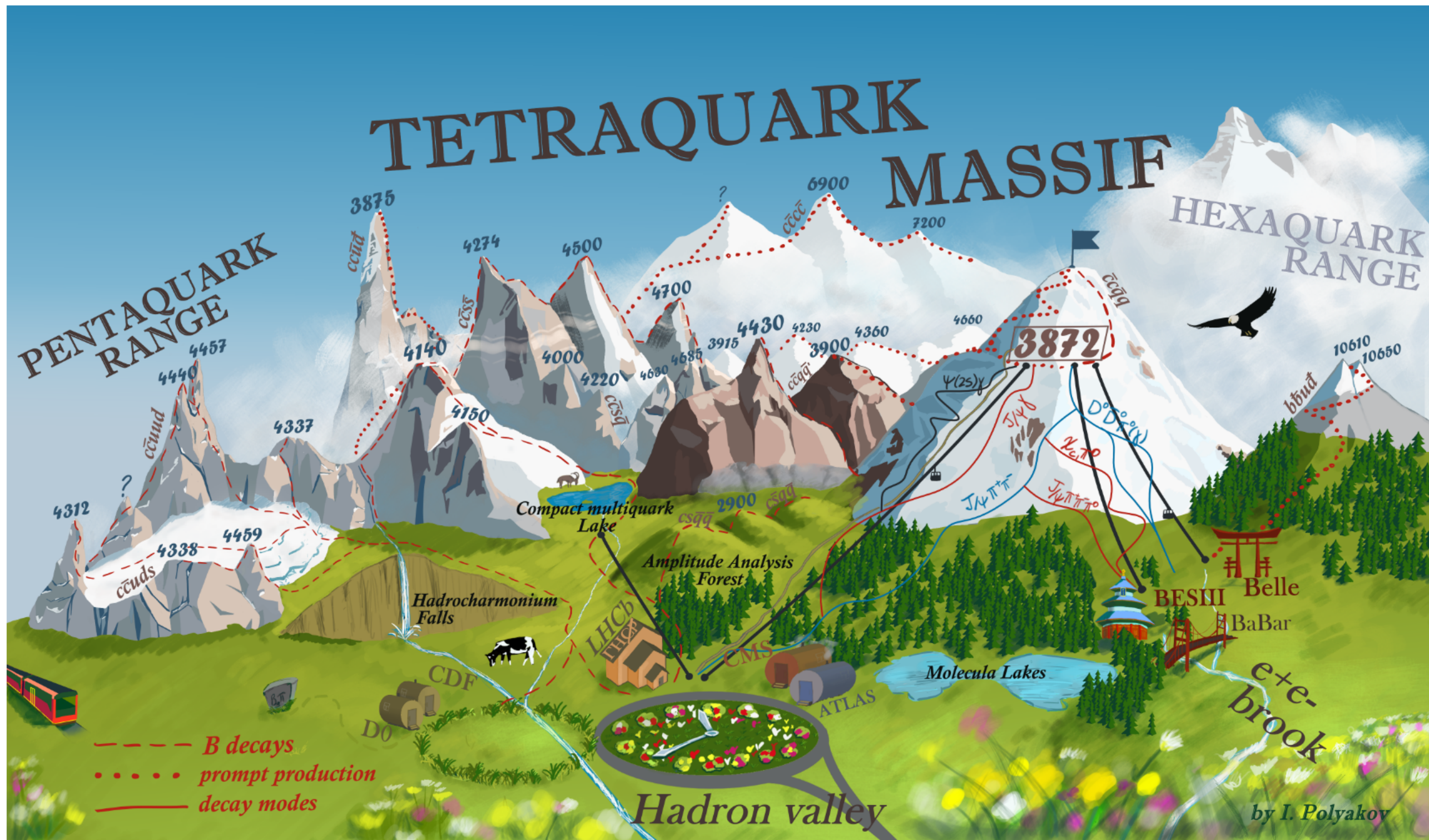
Mass dependence

Line shapes and form factors are model choices, tested by the fit.

Two examples: narrow tetraquarks and pentaquarks

A peak, then a model of the whole decay. Λ^* states alone fail; Pc states are needed. Their nature is still open.

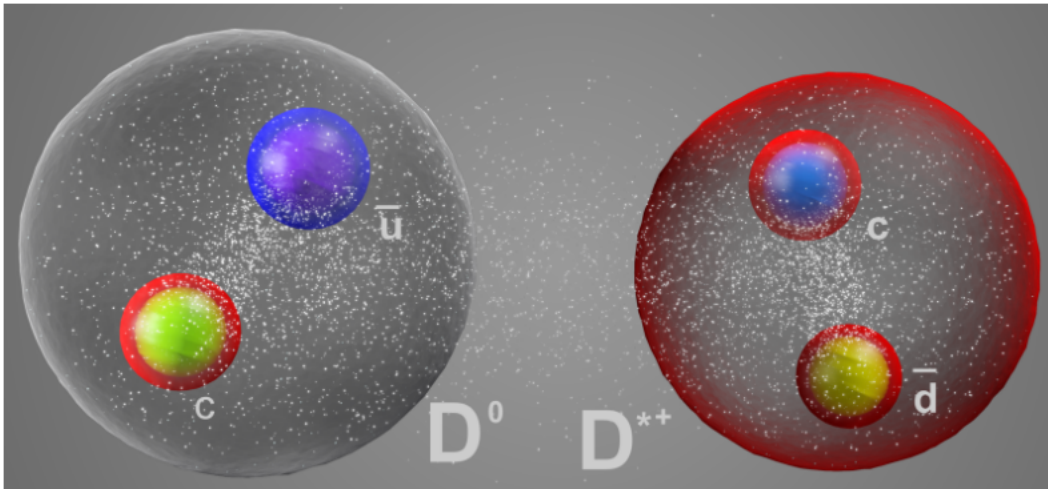
T_{cc}^+ : a narrow peak just below the D^*D threshold, where a Breit–Wigner is not enough.



Backup

T_{cc}^+ structure

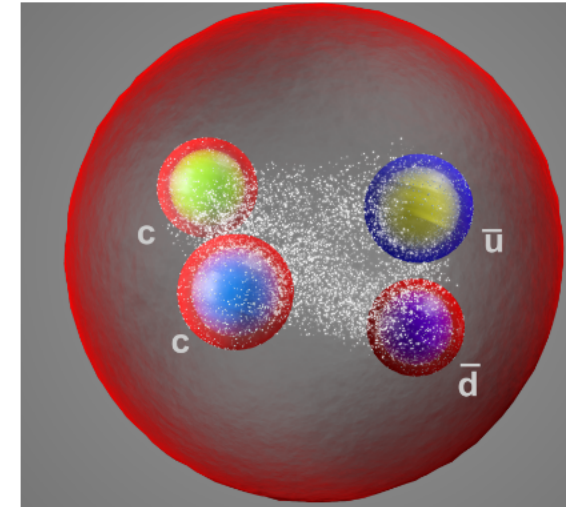
Backup: T_{cc}^+ , two extreme spatial configurations



Molecule: D^0 and D^{*+} held by π , ρ , ... exchange

Molecular configuration

Two mesons, well separated, bound like van der Waals forces. Entirely coupled to $D^{*+}D^0$; the lifetime is limited by the D^{*+} . Spatially extended.



Compact: a (cc) core with $\bar{u} \bar{d}$

Compact configuration

A genuine QCD state with a compact (cc) core, about 1 fm. The lifetime depends on how strongly it couples to the continuum.